MATH 5320 Test 2: Practice

Know Quiz 6 and

1 Continuity

1. Show $f(x) = x^2 + 1$ is continuous at $x = 3$ (using the $\varepsilon - \delta$ definition).
2. Show $f(x) = x^2 + 1$ is continuous where $f : \mathbb{R} \to \mathbb{R}$ (using the $\varepsilon - \delta$ definition).
3. Show $f(x) = x^2 + 1$ is uniformly continuous at $f : [-10, 7] \to \mathbb{R}$ (using the $\varepsilon - \delta$ definition).
4. State the IVT and the EVT.
5. 3.4: 8 (page 169)

2 Differentiability

6. Compute the derivatives of the following using the definition:
   (a) $f(x) = x^2$
   (b) $f(x) = \begin{cases} 3x - 2 + \frac{x^2}{|x|} & : x \neq 0 \\ -2 & : x = 0 \end{cases}$ at $x = 0$.
   (c) $f(x) = \begin{cases} x \sin(\frac{1}{x}) & : x \neq 0 \\ 0 & : x = 0 \end{cases}$ at $x = 0$.
   (d) $f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & : x \neq 0 \\ 0 & : x = 0 \end{cases}$ at $x = 0$.
   (e) $f(x) = \begin{cases} \sin(\frac{1}{x}) & : x \neq 0 \\ 0 & : x = 0 \end{cases}$ at $x = 0$.
7. Prove if $f(x)$ and $g(x)$ are differentiable at $x = c$ then $[f(c)g(c)]' = f'(c)g(c) + f(c)g'(c)$.
8. Prove if f is differentiable at $x = c$ then f is continuous at $x = c$.
9. State the MVT.
10. Find all values of c from the MVT for the following

(a) $f(x) = 3x^2 + 5x + 7$; $[1, 7]$

(b) $f(x) = 3x^2 + 5x + 7$; $[a, b]$

(c) $f(x) = |x|$; $[1, 7]$

(d) $f(x) = |x|$; $[-1, 7]$

11. 4.3:21

12. State Taylor's Theorem

13. Use Taylor's Theorem (n=3) to find a polynomial to apprximate the following functions at $a = 0$. Bound the remainder term for values in the interval $[0, 1]$.

    (a) $f(x) = \sin(2x)$.

    (b) $f(x) = \cos(3x)$.

    (c) $f(x) = e^{5x}$.

3 Integration

14. Prove (using Riemann Sums) that

$$f(x) = \begin{cases} 2 & : x > 2 \\ -3 & : x \leq 2 \end{cases}$$

is integrable over the interval $[0, 3]$. What is that integral?

15. Let $f(x) = x^2+1$, $[a, b] = [1, 4]$ and let $\mathcal{P} = \{1, 2, 3, 3.5, 3.7, 4\}$ be a artition for [1,4] and let $\mathcal{S} = \{1, 2.2, 3.1, 3.6, 4\}$ be a sampling. Compute $RS(f, \mathcal{P}, \mathcal{S})$, $US(f, \mathcal{P})$ and $LS(f, \mathcal{P})$.

16. State the FTC v0, the FTC v1, the FTC v2 and the MVTI.

17. What does the MVTI say about

    (a) $f(x) = x^2 + 1$ over $[a, b] = [-1, 3]$.

    (b) $f(x) = |x|$ over $[a, b] = [-3, 3]$.

18. 5.2:2,3,4

19. 5.4:9,10

20. Let $f_n(x) = x^n$ where $n \in \mathbb{N}$.

(a) Graph $f_n(x)$ on the interval $[0, 1]$ for several values of n until you see the pattern. Explain the pattern.

(b) Compute $\int_0^1 f_n$.

(c) Find the limit $\displaystyle\lim_{n\to\infty} \int_0^1 f_n$.