MATH 5320 Final Exam: Practice

1 Other Stuff

1. Test 1
2. Test 1.1
3. Test 2
4. Test 2 Review

2 Sequences

5. Prove using the definition that
   (a) $\lim_{n\to\infty} \dfrac{k}{n} = 0$ for any $k \in \mathbb{R}$
   (b) $\lim_{n\to\infty} \dfrac{3n+1}{n+2} = 3$
6. Assume $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$. Show $\lim_{n\to\infty} a_n b_n = ab$.
7. Assume $\lim_{n\to\infty} a_n = 0$ and and assume the sequence (b_n) is bounded. Show
   $$\lim_{n\to\infty} a_n b_n = 0.$$
8. Assume $\lim_{n\to\infty} a_n = 0$ and and assume the sequence (b_n) is not bounded. Show $\lim_{n\to\infty} a_n b_n$ is not necessarily zero. That is find (a_n) and (b_n) where $a_n \to 0$ but $a_n b_n \not\to 0$.
9. Prove If (a_n) is convergent then (a_n) is bounded.
10. Use the Monotone Convergence Theorem to show (a_n) as described below has a limit. Compute that limit.
    (a) $a_1 = 1, a_{n+1} = 1 - \frac{1}{a_n+2}$
    (b) $a_1 = 1, a_{n+1} = \sqrt{a_n + 1}$
11. Show the following sequences diverge to infinity.
    (a) $\lim_{n\to\infty} 3n - 1 = \infty$
    (b) $\lim_{n\to\infty} \frac{n+5}{\sqrt{n+1}} = \infty$

3 Limits of Functions

12. prove using the $\varepsilon - \delta$ definition of a limit

$$\lim_{x \to -2} 3x - 1 = -7 \text{ and } \lim_{x \to 3} x^3 - 8 = 19 \text{ and } \lim_{x \to -2} \frac{1}{1+x^2} = \frac{1}{5}$$

$$\lim_{x \to 3} x^2 - 2x = 3 \text{ and } \lim_{x \to 1} \sqrt{x+3} = 2$$

13. If $\lim_{x \to c} f(x) = F$ and $\lim_{x \to c} g(x) = G$ then show $\lim_{x \to c} f(x)g(x) = FG$.

14. If $\lim_{x \to c} f(x) = F$ and $\lim_{x \to c} g(x) = G$ then show $\lim_{x \to c} f(x) + g(x) = F + G$.

15. If $\lim_{x \to c} f(x) = F$ and let $k \in \mathbb{R}$ then show $\lim_{x \to c} kf(x) = kF$.

4 De'Moivre

16. Compute the Taylor Series with $a = 0$ for the following
    (a) f(x) = $\sin(x)$
    (b) f(x) = $\cos(x)$
    (c) f(x) = e^x
    (d) f(x) = $e^{i\theta}$ use Problem 16c

17. State and prove DeMoivre's Theorem.

18. Find trigonometric identities for
    (a) $\cos(2x)$
    (b) $\sin(2x)$
    (c) $\cos(3x)$
    (d) $\sin(3x)$
    (e) $\cos(4x)$
    (f) $\sin(4x)$