Name:_____

MA 5320 Quiz 6

1. Show $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = x^2$ is continuous at $x = 3$.
2. Show $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = x^2$ is continuous.
3. A function has a **removeable discontinuity** at a point $x = a$ if f is discontinuous at $x = a$ and we can redefine $f(a)$ so that f becomes continuos at $x = a$. In each part decide wheteher the unction hasa removeable discontinuoty at the given point. If so explain how to remove it. If not explain why.
   (a) $f(x) = \frac{1}{x}$; $a = 0$
   (b) $f(x) = \frac{x^2-4}{x+2}$; $a = -2$
   (c) $f(x) = x \sin(\frac{1}{x})$; $a = 0$
4. Assume $f : I \to \mathbb{R}$ is bounded on I, an open interval in $\mathbb{R}$.
   (a) Show $g(x) = xf(x)$ is continuous t $x = 0$
   (b) Show $g(x) = xf(x)$ is continuous t $x = a$ for $a \neq 0$ if and only if $f(x)$ is continuous t $x = a$.
5. State the EVT and the IVT.
6. What does the EVT say about the following functions
   (a) $f : (0, 1] \to \mathbb{R}$ continuous
   (b) $f : [0, 1] \to \mathbb{R}$ continuous
7. What does the IVT say about the following function $p(x) = x^5 + x^3 + 1$. Hint: what are $p(1)$ and $p(-1)$?
8. Show f is uniformly continuous: $f : [-10, 5] \to \mathbb{R}$ where $f(x) = x^2$.
9. Show f is **not** uniformly continuous: $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = x^2$.
10. Is the following function is uniformly continuous? Prove or disprove.

$$f : \mathbb{R} \to \mathbb{R} \text{ where } f(x) = x^2.$$