

Name: _____

MA 5320 Quiz 5

1. State the Monotone Convergence Theorem
2. Prove the sequence (a_n) , defined below, converges and find its limit.

$$a_1 = 1 \text{ and } a_{n+1} = 7 - \frac{36}{a_n + 6}$$

- (a) Show (a_n) is increasing.
 - (b) Show (a_n) is bounded above.
 - (c) Quote the MCT.
 - (d) Find the limit.
3. Prove $\lim_{x \rightarrow 3} (x - 3) \sin(x) = 0$
 - (a) prove using the $\varepsilon - \delta$ definition.
 - (b) prove using the squeeze theorem.
 4. Prove: The limit of a sequence is unique.
 5. Prove $\lim_{x \rightarrow 2} \frac{x}{(x-2)^2} = \infty$.
 6. Define the function $f : \mathbb{R} \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} x \sin(\frac{1}{x}) & : x \neq 0 \\ 0 & : x = 0 \end{cases}$$

- (a) Graph f .
- (b) Prove $\lim_{x \rightarrow 0} f(x) = 0$. Quote the theorems you use.