Math 3520 - Test 2

Name:_____

1. Do **one** of the following:
   - Show the sets have the same cardinality $[0, 1)$ and $(2, 3]$.
   - Show the set of irrationals is uncountable.

2. Let $a, b, c \in \mathbb{Z}$ where $\gcd(a, b) = 1$. If $a|c$ and $b|c$ then $ab|c$.

3. Prove $\sqrt{3}$ is irrational.

4. For the following pair of numbers, find their gcd and and find a linear combination of the numbers equal to their gcd. $a = 189$ and $b = 333$

5. Let $(G, *)$ be a group prove
   If $a * a = e$ for all $a \in G$ then G is Abelian.

6. For the following algebraic structure write the operation table. State if the structure is a group or not. If not state which property fails and how. Identify the identity. $(D, \circ)$ where $D \subseteq S_3$ defined as

$$D = \{\sigma_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}\}$$

7. We showed in class that some groups are "the same" by finding a function from $(Z_5^*, \cdot)$ to $(Z_4, +)$ that preserved the operation. Find the function and verify that it preserves the operation. Hint: I defined a function $f : Z_5^* \to Z_4$ where $f(2) = 1$ and $f(4) = 3$

   (a) Define the funtion

   (b) Only verify the following: $f(2 \cdot 3) = f(2) + f(3)$ and $f(4 \cdot 4) = f(4) + f(4)$.