Math 3520 - Test 1 Review

Be certain to **know** the quizzes and . . .

1 Preliminaries

1. Induction, definition of odd, even, sets union, intersection complement
2. Prove $8|5^{2n} - 1$ for all $n \in \mathbb{N}$.

2 Relations

3. Define relation, equivalence relation, well ordered, reflexive, symmetic, trasitive, domain, codomain, partirion of a set
4. We define the given relation from A to B by
$$R = \{(1, a), (2, a), (3, a), (4, a), (1, a), (2, a)\}$$
where $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. What are the domain and codomain?
5. We define the given relation on A by
$$R = \{(1, 1), (2, 1), (3, 1), (4, 1), (1, 2), (2, 2)\}$$
where $A = \{1, 2, 3, 4\}$.
   (a) What are the domain and codomain?
   (b) Is R reflexive? If it is not reflexive, expand R so that it is reflexive.
   (c) Is R symmetric? If it is not symmetric, expand R so that it is symmetric.
   (d) Is R transitive? If it is not transitive, expand R so that it is transitive.
   (e) Is R an equivalense relation?
6. We define the given relation on A by
$$R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 2), (2, 3), (a, b)(c, d)\}$$
where $A = \{1, 2, 3, 4\}$.

(a) Assume R is an equivalence relation. Find (a, b) and (c, d).

(b) What are the equivalence classes for R.

7. We define the given relation on $\mathbb{Z}$ by

$$aRb \Leftrightarrow 4|3a - b.$$

   (a) Prove R is reflexive.

   (b) Prove R is symmetric.

   (c) Prove R is transitive.

   (d) What are the equivalence classes for R.

3 Functions

8. Define functiom, injective, surjective, domain, codomain, range, inverse image, inverse function, permutations

9. Define $f : \{1, 2, 3\} \to \{4, 7, 9\}$ by $f(1) = 4$, $f(2) = 4$ and $f(3) = 9$.

   (a) Is f injective, surjective or bijective? Compute

   (b) Compute $f(\{1, 2\})$, $f^{-1}(\{1, 4\})$ and $f \circ f^{-1}(\{4, 9\})$.

10. Define $f : \mathbb{Z} \to \mathbb{Z}$ by $f(n) = 2n - 1$. Is f injective, surjective or bijective? Prove or disprove.

11. Define $f : (-\infty, 0) \to [0, \infty)$ by $f(x) = x^2$.

    (a) Is f injective, surjective or bijective? Prove or disprove.

    (b) Compute $f((-2, 2))$, $f^{-1}((-2, 2))$, $f^{-1} \circ f(\{4, 9\})$ and $f \circ f^{-1}(\{4, 9\})$.

12. Define $f : \mathbb{R} \setminus \{2\} \to \mathbb{R} \setminus \{1\}$ by $f(x) = \frac{x}{x-2}$. Is f injective, surjective or bijective? Prove or disprove.

13. Let $f : A \to B$ and $g : B \to C$. Prove the following.

    (a) If f and g are injective then $g \circ f$ is injective.

    (b) If f and g are surjective then $g \circ f$ is surjective.

14. Find all bijective functions from $\{1, 2, 3\}$ to $\{1, 2, 3\}$. How many functions did you come up with?

15. List all elements of the set $\mathcal{S}_3$. How many elements are in the set $\mathcal{S}_6$?

16. For the following permutaions:
$\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$ and $\sigma_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}$
compute:

    (a) $\sigma_1 \circ \sigma_1$

    (b) $\sigma_1 \circ \sigma_2 \circ \sigma_2$

    (c) σ_1^3

    (d) σ_1^{-1}

4 Cardinality

17. Define cardinality. That is $A \sim B$ if and only if ...

18. Know $\sim$ is an equivalence relation and what that means.

19. Show $\mathbb{N} \sim 2\mathbb{N}$

20. Show $\mathbb{N} \sim \mathbb{Z}$

21. Let $A = [0, 2]$ and $B = [-1, 6]$. Show $f : A \to B$ given by $f(x) = \frac{7}{2}x - 1$ is a bijection. What does this tell uas about the sets A and B?