

Math 3520 - Test 1

Name: _____

1. Prove for all $n \in \mathbb{N}$ then $3|n^3 - n$.

2. We define the given relation on $\mathbb{Z}$ by

$$aRb \Leftrightarrow 4|a - b.$$

- (a) Prove R is reflexive.
- (b) Prove R is symmetric.
- (c) Note R is an equivalence relation. What are the equivalence classes for R ?

3. Let $f : B \rightarrow C$ and $g : A \rightarrow B$. Prove if f is injective and g is injective then $f \circ g$ is injective.

4. List all elements of the set $\mathcal{S}_2$. How many elements are in the set $\mathcal{S}_5$?

5. For the following permutaions:

$$\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix} \text{ and } \sigma_3 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \end{pmatrix}.$$

compute:

(a) $\sigma_1 \circ \sigma_2 \circ \sigma_3$

(b) σ_1^3

(c) σ_3^{-1}

6. Let $A = (0, \infty)$ and $B = (0, 1)$. Show $f : A \rightarrow B$ given by $f(x) = \frac{1}{x+1}$ is a bijection. What does this tell us about the sets A and B ?