Math 3520 - Final Exam Review

Prepare

- Test 1
- Test 2
- Test 2 Review
- and this Review

1 New Stuff

1. Definition of a **subgroup** and of an **isomorphism**.
2. For the group $(\mathbb{Z}, +)$ prove, using the 2-step subspace test, that $H = \{3n : n \in \mathbb{Z}\}$ is a subgroup.
3. For the group $(\mathbb{Z}, +)$ Show $H = \{2n + 1 : n \in \mathbb{Z}\}$ is not a subgroup.
4. For the group $(S_3, \circ)$ Show

$$H = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \right\}$$

   is not a subgroup of S_3.
5. Let G be any group and let g be a fixed element of G then prove, using the 2-step subspace test, that $H = \{gag^{-1} | a \in G\}$ is a subgroup.
6. Let G be any group then prove, using the 2-step subspace test, that $H = \{a \in G | ag = ga \, \forall g \in G\}$ is a subgroup.
7. For the groups $G_1 = (\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2, +)$ and $G_2 = (\mathbb{Z}_8, +)$
   (a) Find the orders of the elements $(1, 1, 1)$ and $(1, 0, 1)$ in G_1 and the orders of the elements 1, 2, 3 in G_2?
   (b) Are any of the above elements generators for their respective groups?
   (c) Why aren't the groups isomorphic?
8. For the groups $G_1 = (\mathbb{Z}_9^*, +)$ and $G_2 = (Z_6, +)$

(a) Find the orders of three diffferent non identity elements in G_1 and the orders of the elements 1, 2, 3 in G_2?

(b) Are any of the above elements generators for their respective groups?

(c) The two groups are isomorphic. Find the isomorphism $f : G_1 \to G_2$.

9. Note that $(G, \cdot)$ is a group where $G = \{2^n : n \in \mathbb{Z}\}$ and $\cdot$ is regular multiplication. Prove Axioms G_1 and G_3 for $(G, \cdot)$.

10. Note that $(G, \cdot)$ is a group where $G = \{2^n : n \in \mathbb{Z}\}$ and $\cdot$ is regular multiplication. Show $G \cong \mathbb{Z}$ where $\mathbb{Z}$ is a group over addition. I used the isomorphism $f : \mathbb{Z} \to G$. We need to prove f preserves the operation and that f is a bijection.

2 Relations

11. We define the given relation on A by

$$R = \{(1,1),(2,1),(3,1),(4,1),(1,2),(2,2)\}$$

where $A = \{1,2,3,4\}$.

(a) What are the domain and codomain?

(b) Is R reflexive? If it is not reflexive, expand R so that it is reflexive.

(c) Is R symmetric? If it is not symmetric, expand R so that it is symmetric.

(d) Is R transitive? If it is not transitive, expand R so that it is transitive.

(e) Is R an equivalense relation?

12. We define the given relation on A by

$$R = \{(1,1),(2,2),(3,3),(4,4),(1,2),(2,1),(3,2),(2,3),(a,b)(c,d)\}$$

where $A = \{1,2,3,4\}$.

(a) Assume R is an equivalence relation. Find (a,b) and (c,d).

(b) What are the equivalence classes for R.

13. We define the given relation on $\mathbb{Z}$ by

$$aRb \Leftrightarrow 4|3a-b.$$

(a) Prove R is reflexive.
(b) Prove R is symmetric.
(c) Prove R is transitive.
(d) What are the equivalence classes for R.

3 Functions

14. Define $f:\{1,2,3\}\to\{4,7,9\}$ by $f(1)=4$, $f(2)=4$ and $f(3)=9$.

(a) Is f injective, surjective or bijective? Compute
(b) Compute $f(\{1,2\})$, $f^{-1}(\{1,4\})$ and $f\circ f^{-1}(\{4,9\})$.

15. Define $f:(-\infty,0)\to[0,\infty)$ by $f(x)=x^2$.

(a) Is f injective, surjective or bijective? Prove or disprove.
(b) Compute $f((-2,2))$, $f^{-1}((-2,2))$, $f^{-1}\circ f(\{4,9\})$ and $f\circ f^{-1}(\{4,9\})$.

16. Define $f:\mathbb{R}\setminus\{2\}\to\mathbb{R}\setminus\{1\}$ by $f(x)=\frac{x}{x-2}$. Is f injective, surjective or bijective? Prove or disprove.

17. Let $f:A\to B$ and $g:B\to C$. Prove the following.

(a) If f and g are injective then $g\circ f$ is injective.
(b) If f and g are surjective then $g\circ f$ is surjective.

4 Cardinality

18. Define cardinality. That is $A\sim B$ if and only if ...

19. Know $\sim$ is an equivalence relation and what that means.

20. Show $\mathbb{N}\sim 2\mathbb{N}$

21. Show $\mathbb{N}\sim\mathbb{Z}$

22. Let $A=[0,2]$ and $B=[-1,6]$. Show $f:A\to B$ given by $f(x)=\frac{7}{2}x-1$ is a bijection. What does this tell uas about the sets A and B?