Measure Theory - Test 1

Name:_____

1. State the following definitions
   (a) Let S be a set. We say $\mathcal{S}$ is a σ-algebra on S iff
   (b) S is a null set iff
   (c) Define the outer Lebesgue measure.
   (d) $E \in \mathcal{M}$ iff and only if
   (e) On $\mathbb{R}$ with the usual topology. Define the Borel sets.
2. Let A and B be null sets in $\mathbb{R}$. Show $A \cup B$ is null.
3. Let A and B be measureable sets in $\mathbb{R}$. Show $A \cup B$ is measureable. Hint: You should use the subadditivity of $m*$.
4. Do one of the following:
   - If $m^*(A) = 0$ then $m^*(AUB) = m^*(B)$.
   - If $m^*(A \triangle B) = 0$ then $m^*(A) = m^*(B)$.
5. Let $\mathcal{A}_1$ be the σ-algebra generated by the open intervals in $\mathbb{R}$ and Let $\mathcal{A}_2$ be the σ-algebra generated by the closed intervals in $\mathbb{R}$. Show $\mathcal{A}_1 = \mathcal{A}_2$.
6. Show the following are equivalent:
   - $E \in \mathcal{M}$.
   - For all $\varepsilon > 0$ there is an open set O so that $E \subseteq O$ and $m^*(O/E) < \varepsilon$.
7. Assume A and B are independent events. Show A^c and B are independent events.