Math 6250: Test 1 Review

First you should be able to complete Quiz 1, 2, 3 and 4.

1 Foundations

1. State the Peano Axioms.
2. Use induction to prove something simple.
3. Define equivalence class.
4. Be able to show a given relation is an equivalence class.
5. Be able to show a given relation with an operation is well defined. Know what well defined means.
6. Define $(\mathbb{Z}, +, \cdot)$ using equivalence class and show the operations are well defined.
7. Define $(\mathbb{Q}, +, \cdot)$ using equivalence class and show the operations are well defined.

2 Functions and Cardinality

1. Know the definition of injective and surjective.
2. Prove a statement like: If f is injective and g is injective then $f \circ g$ is injective.
3. Show a given function is injective or surjective.
4. Prove $\mathbb{Q} \sim \mathbb{N}$, $\mathbb{Z} \sim \mathbb{N}$, $\mathbb{R} \not\sim \mathbb{N}$,

3 The Real and the Complex Numbers

1. State the definition of the Reals.
2. State the definition of a Field.
3. Compute a sup or inf.
4. Prove: Let $\alpha = \sup(A)$. If $\varepsilon > 0$ then there is some $x \in A$ so that $\alpha - \varepsilon < x \leq \alpha$.

5. Prove: Let $\alpha = \sup(A)$. If $\alpha \notin A$ then A is infinite.

6. State and use the nested interval property.

7. Solve expressions like: $x^4 = 1$, $x^3 = 2$ and $x^4 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$.

4 Sequences

1. Define convergent and Cauchy.

2. Show a given sequence is convergent or Cauchy (or not).

3. Prove if a sequence is convergent then it is bounded (similarly for Cauchy sequences).

4. Know how to prove a property like: If (a_n) and (b_n) are convergent then $(a_n + b_n)$ is convergent (we learned for properties like this).

5. Prove and state the Monotone convergence Theorem.

6. State and use the Bolzano-Weierstrauss Theorem.