Math 6250: Test 1

Name:____

1. Do one of the following.

   - State the Peano Axioms
   - Prove $1+r^1+r^2+\cdots+r^n = \frac{1-r^{n+1}}{1-r}$ using induction and compute $1+(\frac{1}{2})^1+(\frac{1}{2})^2+(\frac{1}{2})^3+\cdots++(\frac{1}{2})^{100}$ and compute $1+(\frac{1}{2})^1+(\frac{1}{2})^2+(\frac{1}{2})^3+\cdots$.

2. Do one of the following

   (a) State the definition of the set $\mathbb{Z}$ in terms of an equivalence relation (as in class). And state the definition of addition and multiplication.

   (b) State the definition of the set $\mathbb{Q}$ in terms of an equivalence relation (as in class). And state the definition of addition and multiplication.

   (c) State the definition of the set $\mathbb{R}$ and define the words complete, ordered and field.

3. Look at the sets A and B. If $A \sim B$ prove it. If not state it.

   (a) $A = \mathbb{N}$ and $B = \mathbb{Z}$

   (b) $A = \mathbb{Q}$ and $B = \mathbb{R}$

4. Prove the sequence $(\frac{5n+1}{3n+2})$ is convergent.

5. The following questions refer to $\mathbb{C}$.

   (a) Find all solutions in $\mathbb{C}$ to the following equation.

   $$x^3 = -1$$

   (b) Use DeMoivre's equation to prove the trigonometric following identity.

   $$\cos(3\theta) = \cos^3(\theta) - 3\cos(\theta)\sin^2(\theta)$$

6. State the Monotone Convegent Theorm and use it to prove the following sequence is convergent. And find its limit.

$$a_1 = 12 \text{ and } a_{n+1} = \sqrt{4 + a_n}$$

7. Show the sequence defined below is not Cauchy.
$S_1 = \sum_{k=1}^{1} \frac{1}{k} = \frac{1}{1}$, $S_2 = \sum_{k=1}^{2} \frac{1}{k} = \frac{1}{1} + \frac{1}{2}$, $S_3 = \sum_{k=1}^{3} \frac{1}{k} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3}$
and $S_n = \sum_{k=1}^{n} \frac{1}{k} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$

That is show (S_n) is not Cauchy.

8. Show the sequence defined below is convergent.
   $S_n = \sum_{k=1}^{n} \frac{1}{k^2-k}$