Math 6250 Quiz 5

Name:_____

1. Prove the following sequence has a limit and prove it.

   $a_1 = 2$, $a_2 = 2 + \frac{1}{2}$, $a_3 = 2 + \frac{1}{2+\frac{1}{2}}$,

   $a_3 = 2 + \frac{1}{2+\frac{1}{2+\frac{1}{2}}}$, ...

2. For the following two sequences find an nth term representation.

   (a) $a_1 = 3$ and $a_{n+1} = -1 + 2a_n$.

   (b) $a_1 = 1$, $a_2 = 2$ and $a_{n+2} = 2a_n + a_{n+1}$. Refer to our Fibonacci sequence example.

3. For the following functions find their Taylor series representation at $x = a$ from the formula. Also find the radius of convergence.

   (a) $f(x) = e^{2x}$ at $a = 1$

   (b) $f(x) = \frac{1}{1-2x}$ at $a = 1$

4. For the following functions find the Taylor series representation at $x = 0$. Find the minimal number of terms to approximate the series to 10^{-5}.

   (a) $f(x) = \sin(x)$

   (b) $f(x) = \tan^{-1}(x)$. Then use the series to approximate π

5. Define the sequence below for some x as

   $$x, x^x, x^{x^x}, x^{x^{x^x}}, \ldots$$

   If limit of this sequence is 2 then compute x.

6. My nephew has grown since I last saw him. He is now able to climb the stairs and he can even leap two steps at once (a feat he is very proud of). He noticed that on a staircase of 5 steps he has 8 different ways he can climb the stairs: they are

   - 1 step, 1 step, 1 step, 1 step, 1 step
   - 1 step, 1 step, 1 step, 2 steps
   - 1 step, 1 step, 2 steps, 1 step

- 1 step, 2 steps, 1 step, 1 step
- 2 steps, 1 step, 1 step, 1 step
- 2 steps, 2 steps, 1 step
- 2 steps, 1 step, 2 steps
- 1 step, 2 steps, 2 steps

On a staircase of 4 steps their are 5 different ways he can climb the staircase. Find a formula to help my nephew to determine the number of ways he can climb a staircase of n steps. Prove it.

7. We have seen the below formula several times. Prove it in three ways.

$$1 + r + r^2 + r^3 + r^4 + \cdots$$

(a) Prove it with algebra.

(b) Prove it with induction.

(c) Prove it be computing the Taylor polynomial for $f(x) = \frac{1}{1-x}$.