Math 6250 Quiz 2

Name:____

1. Show $f : (0, 1) \to (1, \infty)$ defined by $f(x) = \frac{1}{x}$ is bijective.

2. For the following find a bijective function from one set to the other (except for 2c). Demonstrate it is bijective.
   (a) $\mathbb{N} \sim \mathbb{Z}$
   (b) $\mathbb{N} \sim \mathbb{Q}$
   (c) $\mathbb{N} \not\sim \mathbb{R}$
   (d) $(0, 1) \sim [0, 1)$. I think this one is tricky.

3. Show if $f : A \to B$ is bijective and $g : B \to C$ is bijective then $g \circ f : A \to C$ is bijective.

4. Let A, B be nonempty bounded subsets of $\mathbb{R}$. Define $A+B = \{a+b|a \in A, b \in B\}$ $-A = \{-a|a \in A\}$
   (a) For $A = (1, 3)$ and $B = [-4, -1]$. Compute $A + B$ and $-A$.
   (b) For $A = (1, 3)$ and $B = [-4, -1]$. Compute $\sup(A)$, $\sup(B)$, $\sup(A + B)$ and $\sup(-A)$.
   (c) Prove the following fact.
   For any A, B nonempty bounded subsets of $\mathbb{R}$ we have that
   $$\sup(A) + \sup(B) = \sup(A + B).$$
   (d) Guess a similar fact about the $\sup(-A)$.

5. Prove the triangle inequality. That is for all $x, y \in \mathbb{R}$
   $$|x + y| \leq |x| + |y|.$$
   Hint: It is easier to show $|x + y|^2 \leq (|x| + |y|)^2$ by looking at various cases.

6. How have we defined the reals? The reals are also the only complete ordered field. What are the definitions for 1. complete. 2. ordered and 3. field. Look these up.

7. Prove the sup of set is unique.