Math 3330 - Test 2

Name:____

1. Let $\mathbf{r}(t) = \langle 2\cos(t), 3\sin(t) \rangle$.

   (a) Graph.

   (b) Compute the tangent line at the point $t = 3\pi/4$.

   (c) Compute the velocity and acceleration $t = \pi/4$. Add these two to your graph.

2. Let $f(x, y) = x^2 + y^2$.

   (a) Graph the contour plot for values $z = -1, 0, 1, 2, 3$.

   (b) Compute the gradient at the point $P(1, -1)$, place the gradient on your graph.

   (c) Compute the directional derivative of $f(x, y)$ at the point P in the direction of $\mathbf{v}\langle 1, 2\rangle$.

3. Let $f(x,y,z) = e^{x^2-yz^2} + 3x - 2yz$. Let P be the point $(2,1,-2)$.

   (a) Compute the tangent plane of $f(x,y,z)$ at P.

   (b) Approximate the value $f(1.9, 1.1, -2.1)$ using your plane.

4. Using the second Derivative Test find and classify the extrema of the following function: $f(x, y) = x^2 + 2x + y^3 - 4y^2 + 4y + 2$

   (a) If $D(P) > 0$ and $f_{xx}(P) > 0$ then f has a local minimum at P.
   (b) If $D(P) > 0$ and $f_{xx}(P) < 0$ then f has a local maximum at P.
   (c) If $D(P) < 0$ then f has a saddle point at P.
   (d) If $D(P) = 0$ then the second derivative test is inconclusive.

   where $D(x, y) = f_{xx}(x, y)f_{yy}(x, y) - (f_{xy}(x, y))^2$.

5. Using the La Grange Multipliers find the Max or Minimum of the following function subject to the given constraint: $f(x, y, z) = x^2 + y^2 + z^2$ subject to $2x - y + z = 4$

6. $\iint_R \sin(y^2)\, dA$ over the region contained within $y = x$, $y = -2x$ and $y = 4$.

7. Prove: If $\|\mathbf{r}(t)\|$ is constant then $\mathbf{r}(t)$ is perpendicular to $\mathbf{r}'(t)$.