Math 3330 - Review for Test 1

To prepare for the test you should do Quiz 1 and Quiz 2 the homework and here are some sample problems.

1 Introduction to parametric equations

1. for the following functions find a parametrization: $y = x - 2$, $x^2 + y^2 = 4$ and $x = 4$.
2. for the following functions given parametrically find a representation given in rectangular coordinates:
   - $x = 3t - 1$ $\qquad y = t^2 - 2$
   - $x = 3\cos(t)$ $\qquad y = 2\sin(t)$. Hint what is $\frac{x^2}{9} + \frac{y^2}{4}$ equal to?
   - $x = 2e^t$ $\qquad y = 3e^t$
3. Let $x = 2\sin(t)$ $\qquad y = 3\cos(t)$.
   (a) Sketch its graph and indicate the increasing direction of t.
   (b) Compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.
   (c) Find the equation of the line tangent at the point $t = \pi/4$.
4. For the triangle with vertices $P(0,0)$, $Q(1,2)$ and $R(-1,1)$ find a parametrization.
5. Find the arc length for the function over the interval $x = \cos(2t)$ and $y = \sin(2t)$ where $0 \leq t \leq \pi$.

2 Polar

6. Graph: $r = \cos(2\theta)$
7. Graph: $r = 16\sin(2\theta)$
8. Graph: $r = \sin(\theta)$
9. Graph: $r = 1 + \sin(\theta)$
10. Graph: $r = 2 + \sin(\theta)$

11. Graph: $r = e^{\theta}$

12. Translate the following from rectangular to polar: $y = 2x$, $y = 3x - 1$, $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$.

13. Translate the following from polar to rectangular: $r = 2$, $r = 4\cos(\theta)$ and $r = \frac{7}{2\cos(\theta) - 2\sin(\theta)}$.

14. Find $\frac{dy}{dx}$ for $r = e^{\theta}$.

15. Calculate the arc length for the entire graph of: $r = 7\cos(\theta)$

16. Calculate the arc length for the entire graph of: $r = e^{3\theta}$

17. Find the area of the region in the first quadrant and inside the graph $r = e^{2\theta}$ where $0 \leq \pi/2$.

18. Find the area of the region in the first quadrant and inside the graph $r = \sin(2\theta)$ from $0 \leq \pi/2$.

3 Conic Sections

19. Graph the two parabolas
    (a) $y^2 = 4x$
    (b) $x^2 = -8y$

20. Graph the two ellipses
    (a) $(x+3)^2 + 4(y-5)^2 = 16$
    (b) $\frac{1}{4}x^2 + \frac{1}{9}(y+2)^2 - 1 = 0$

21. Graph the two hyperbolas
    (a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$
    (b) $9y^2 - x^2 = 36$

4 These are from quiz 2 and should prepare you for the test

22. Graph the following:

    (a) $z = x^2$ in $\mathbb{R}^3$ - remember first graph in $\mathbb{R}^2$ then in $\mathbb{R}^3$. Your answer should include both graphs.

    (b) $\frac{z^2}{4} - y^2 = 16$ - make certain to label points of intersection on the z-axis and the y-axis.

    (c) $x^2 + y^2 + (z-4)^2 = 2$.

23. Find the vector $\mathbf{v} \in \mathbb{R}^2$ so that $\mathbf{v}$ is 3-units long and if its initial point is the origin, $\mathbf{v}$ makes a 135% angle with the x-axis.

24. Let $\mathbf{v} \in \mathbb{R}^3$ so $\mathbf{w} = \langle 1, 2, 3 \rangle$ that $2\mathbf{v} + \mathbf{w} = 3\mathbf{v}$. Compute the norm of $\|\mathbf{v}\|$.

25. Find a vectors so that

    (a) parallel to $y = x + 2$ and is one unit long (in $\mathbb{R}^2$)

    (b) perpendicular to $y = x + 2$ and is one unit long (in $\mathbb{R}^2$)

    (c) parallel to z-axis and is three units long (in $\mathbb{R}^3$)

    (d) has initial point $P(1, 2, 3, 4)$ and final point $Q(2, -1, 0, -1)$. (in $\mathbb{R}^4$)

    (e) parallel to xz-plane and is two units long (in $\mathbb{R}^3$)

    (f) perpendicular to xz-plane and is two units long (in $\mathbb{R}^3$)

26. Let $\mathbf{v_1}, \mathbf{v_2} \in \mathbb{R}^3$ so $\mathbf{v_1} = \langle 1, 2, 3 \rangle$ and $\mathbf{v_2} = \langle 2, -1, 1 \rangle$. Find c_1 and c_2 so that

$$c_1\mathbf{v_1} + c_2\mathbf{v_2} = \langle 1, -7, -3 \rangle.$$

27. Let $\mathbf{v_1}$, $\mathbf{v_2}$ and $\mathbf{v_3} \in \mathbb{R}^3$ so $\mathbf{v_1} = \langle 1, 2, 3 \rangle$ $\mathbf{v_2} = \langle 2, -1, 1 \rangle$ and $\mathbf{v_3} = \langle 4, -3, 7 \rangle$.

    (a) Find angle between $\mathbf{v_1}$ and $\mathbf{v_2}$.

    (b) Compute $\frac{1}{\|\mathbf{v_1}\|^2}(\mathbf{v_1} \cdot \mathbf{v_1})\mathbf{v_1}$.

    (c) Find three different unit vectors perpendicular to $\mathbf{v_1}$.

    (d) Find $\mathbf{u}$ a unit vector parallel to $\mathbf{v_3}$.

    (e) Compute $\mathbf{u} \cdot \mathbf{v_3}$ and compute $\|\mathbf{v_3}\|$.

28. Prove $\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$.

29. Prove $\|\mathbf{v} + \mathbf{w}\|^2 + \|\mathbf{v} - \mathbf{w}\|^2 = 2\|\mathbf{v}\|^2 + 2\|\mathbf{w}\|^2$.

30. Find the work done by a force given by $\mathbf{F} = 4\imath - 6\jmath + k$ that is applied to a point that moves from $P(1, 2, 3)$ to $Q(2, -1, 3)$.

31. Let $\mathbf{v_1}$, $\mathbf{v_2}$ and $\mathbf{v_3} \in \mathbb{R}^3$ so $\mathbf{v_1} = \langle 1, 2, 3\rangle$ $\mathbf{v_2} = \langle 2, -1, 1\rangle$ and $\mathbf{v_3} = \langle 4, -3, 7\rangle$.

    (a) Compute $\mathbf{v_1} \times \mathbf{v_2}$.

    (b) Compute the area of the parallelogram formed by the two vectors $\langle 2, -1\rangle$ and $\langle 4, -3\rangle$. Also draw the vectors and the parallelogram.

    (c) Compute the volume of a parallelepiped formed by the three vectors $\mathbf{v_1}$, $\mathbf{v_2}$ and $\mathbf{v_3}$.

    (d) Find a single unit vector $\mathbf{w}$ that is simultaneously perpendicular to $\mathbf{v_1}$ and $\mathbf{v_2}$.

    (e) Show that any vector of the form $\langle a - 3b, a, b - a\rangle$ where $a, b \in \mathbb{R}$ is perpendicular to $\mathbf{v_1}$.