Math 3330 - Quiz 2

Name:____

1. Graph the following:
   (a) $z = x^2$ in $\mathbb{R}^3$ - remember first graph in $\mathbb{R}^2$ then in $\mathbb{R}^3$. Your answer should include both graphs.
   (b) $\frac{z^2}{4} - y^2 = 16$ - make certain to label points of intersection on the z-axis and the y-axis.
   (c) $x^2 + y^2 + (z-4)^2 = 2$.

2. Find the vector $\mathbf{v} \in \mathbb{R}^2$ so that $\mathbf{v}$ is 3-units long and if its initial point is the origin, $\mathbf{v}$ makes a 135% angle with the x-axis.

3. Let $\mathbf{v} \in \mathbb{R}^3$ so $\mathbf{w} = \langle 1, 2, 3 \rangle$ that $2\mathbf{v} + \mathbf{w} = 3\mathbf{v}$. Compute the norm of $\|\mathbf{v}\|$.

4. Find a vectors so that
   (a) parallel to $y = x + 2$ and is one unit long (in $\mathbb{R}^2$)
   (b) perpendicular to $y = x + 2$ and is one unit long (in $\mathbb{R}^2$)
   (c) parallel to z-axis and is three units long (in $\mathbb{R}^3$)
   (d) has initial point $P(1, 2, 3, 4)$ and final point $Q(2, -1, 0, -1)$. (in $\mathbb{R}^4$)
   (e) parallel to xz-plane and is two units long (in $\mathbb{R}^3$)
   (f) perpendicular to xz-plane and is two units long (in $\mathbb{R}^3$)

5. Let $\mathbf{v_1}, \mathbf{v_2} \in \mathbb{R}^3$ so $\mathbf{v_1} = \langle 1, 2, 3 \rangle$ and $\mathbf{v_2} = \langle 2, -1, 1 \rangle$. Find c_1 and c_2 so that
$$c_1\mathbf{v_1} + c_2\mathbf{v_2} = \langle 1, -7, -3 \rangle.$$

6. Let $\mathbf{v_1}$, $\mathbf{v_2}$ and $\mathbf{v_3} \in \mathbb{R}^3$ so $\mathbf{v_1} = \langle 1, 2, 3 \rangle$ $\mathbf{v_2} = \langle 2, -1, 1 \rangle$ and $\mathbf{v_3} = \langle 4, -3, 7 \rangle$.
   (a) Find angle between $\mathbf{v_1}$ and $\mathbf{v_2}$.
   (b) Compute $\frac{1}{\|\mathbf{v_1}\|^2}(\mathbf{v_1} \cdot \mathbf{v_1})\mathbf{v_1}$.
   (c) Find three different unit vectors perpendicular to $\mathbf{v_1}$.
   (d) Find $\mathbf{u}$ a unit vector parallel to $\mathbf{v_3}$.
   (e) Compute $\mathbf{u} \cdot \mathbf{v_3}$ and compute $\|\mathbf{v_3}\|$.

7. Prove $\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$.

8. Prove $\|\mathbf{v} + \mathbf{w}\|^2 + \|\mathbf{v} - \mathbf{w}\|^2 = 2\|\mathbf{v}\|^2 + 2\|\mathbf{w}\|^2$.

9. Find the work done by a force given by $\mathbf{F} = 4\imath - 6\jmath + k$ that is applied to a point that moves from $P(1, 2, 3)$ to $Q(2, -1, 3)$.

10. Let $\mathbf{v_1}$, $\mathbf{v_2}$ and $\mathbf{v_3} \in \mathbb{R}^3$ so $\mathbf{v_1} = \langle 1, 2, 3\rangle$ $\mathbf{v_2} = \langle 2, -1, 1\rangle$ and $\mathbf{v_3} = \langle 4, -3, 7\rangle$.

    (a) Compute $\mathbf{v_1} \times \mathbf{v_2}$.

    (b) Compute the area of the parallelogram formed by the two vectors $\langle 2, -1\rangle$ and $\langle 4, -3\rangle$. Also draw the vectors and the parallelogram.

    (c) Compute the volume of a parallelapiped formed by the three vectors $\mathbf{v_1}$, $\mathbf{v_2}$ and $\mathbf{v_3}$.

    (d) Find a single unit vector $\mathbf{w}$ that is simultaneously perpendicular to $\mathbf{v_1}$ and $\mathbf{v_2}$.

    (e) Show that any vector of the form $\langle a - 3b, a, b - a\rangle$ where $a, b \in \mathbb{R}$ is perpendicular to $\mathbf{v_1}$.