Math 2320 - Review for Test 1

To prepare for the test you should do Quiz 1, Quiz 2, Quiz 3, Trig Review, the homework and here are some sample problems. An extremely important section is Section 5

1 Definition of the Integral

1. Compute the Riemann sum approximation of $\int_1^3 x^2 + 1$ with $n = 3$.
2. Compute $\int_1^3 x^2 + 1$ by using the definition of the integral.

2 Fundamental Theorem Part I

3. Compute the following
   (a) $\dfrac{d}{dx}\left[\displaystyle\int_3^x \sin(t^3)\,dt\right]$
   (b) $\dfrac{d}{dx}\left[\displaystyle\int_3^{x^2} \sin(t^2)\,dt\right]$
   (c) $\dfrac{d}{dx}\left[\displaystyle\int_{2x}^{-3x} \sin(t^2)\,dt\right]$
   (d) $\dfrac{d}{dx}\left[x^2\displaystyle\int_0^x \sin(t^2)\,dt\right]$

3 Average Value and the MVT and Motion

4. Compute the Average Value for the following functions:
   (a) $f(x) = 3x^2$ over $[1, 4]$
   (b) $f(x) = 1 + |x|$ over $[-1, 4]$
   (c) $f(x) = 3 + 2x$ over $[0, b]$ where $b \in \mathbb{R}$ and $b > 0$.
5. Find a $c \in [a, b]$ so that $f(c) = f_{ave}$ over the given interval.
   (a) $f(x) = 3x^2$ over $[1, 4]$
   (b) $f(x) = 1 + |x|$ over $[-1, 4]$

(c) $f(x) = 3 + 2x$ over $[0, b]$ where $b \in \mathbb{R}$ and $b > 0$.

(d) $f(x) = x^2$ over $[-4, 1]$

6. Let $a(t) = -32$ feet per second squared. Assume we throw a ball downward at an initial velocity of 16 feet per second from the roof of a 192 foot high building.

   (a) Find the $v(t)$ and $s(t)$ equations.

   (b) At what time does the ball hit the ground?

   (c) How fast is the ball moving when the ball hits the ground?

7. Let $a(t) = 6\sin(2t)$. Assume the object is moving at 12 units per second at time zero and that the object's position is 5 units at time zero (that is $s(0) = 5$.) Find the $v(t)$ and $s(t)$ equations.

4 Area, Volume and Arclength

8. Define the region as contained within the parabola $y = 2x^2$ and below the line $y = 4x + 16$. Find the area of the given region.

9. Define the region as contained within the parabola $y^2 = x$ and the line $y = -x + 2$. Find the area of the given region.

10. Define the region as contained below the function $y = \sin x$ and above the line $y = \frac{2\sqrt{2}}{3\pi}x$ in the first quadrant. Note one of the points of intersection occurs at $P(\frac{3\pi}{4}, \frac{\sqrt{2}}{2})$ Find the area of the given region.

11. Define the region as between the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and the line $y = \frac{3}{2}x - 3$ in the fourth quadrant. — **JUST SET UP THE INTEGRAL - both a dy and a dx Integral**

12. Define the region as contained within the parabola $y^2 = x$ and the line $y = -x + 2$. Find the area of the given region. — **JUST SET UP THE INTEGRAL**

13. Define the region as contained within the parabola $y = 2x^2$ and below the line $y = 4x + 16$. Find the volume of the given region rotated about the x-axis.

14. Define the region as contained within the parabola $y^2 = x$ and the line $y = -x + 2$. Find the volume of the given region rotated about the y-axis.

15. Define the region as contained below the function $y = e^x$ and above the line $y = x$ from $x = 0$ to $x = 4$. Find the area of the given region.

16. Define $y = 3x - 1$. Compute the arclength from $x = 0$ to $x = 3$ using the arclength formula from class:

$$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx.$$

5 Basic Integrals, U-sub and By-Parts

17. $\int 3x - 4 + \frac{5}{6x}\, dx$

18. $\int \cos x \frac{1}{1+x^2}\, dx$

19. $\int \csc(x)\cot(x)\, dx$

20. $\int x\sqrt{x^2+1}\, dx$

21. $\int x^3(x^4-7)^9\, dx$

22. $\int x\sec(x^2+1)\tan(x^2+1)\, dx$

23. $\int \frac{e^x}{1+e^x}\, dx$

24. $\int \frac{e^x}{1+e^{2x}}\, dx$

25. $\int e^x \csc(e^x)\cot(e^x)\, dx$

26. $\int xe^{2x}dx$

27. $\int xe^{3x^2}dx$

28. $\int x^2e^{4x}dx$

29. $\int \arctan(x)dx$

30. $\int x \sin(4x) dx$

31. $\int e^x \sin(4x) dx$