TRIG. INTEGRALS

① $\int \sin^2(3x)\,dx$

$= \int \frac{1}{2}\left[1 - \cos(6x)\right] dx$

$= \boxed{\frac{1}{2}\left[x - \frac{1}{6}\sin(6x)\right] + C}$

② $\int \sin^2(3x)\cos^3(3x)\,dx$

Cosine has the odd power. So u will be sine.

$= \int \sin^2(3x)\cdot\cos^2(3x)\cdot\cos^1(3x)\,dx$

$= \int \sin^2(3x)\left[1 - \sin^2(3x)\right]\cos(3x)\,dx$

$u = \sin(3x)$

$du = 3\cos(3x)\,dx$

$\frac{1}{3}du = \cos(3x)\,dx$

$= \int u^2\left[1 - u^2\right]\frac{1}{3}du$

$= \frac{1}{3}\int u^2 - u^4\,du = \frac{1}{3}\left(\frac{u^3}{3} - \frac{u^5}{5}\right) + C$

$= \boxed{\frac{1}{3}\left(\frac{\sin^3(3x)}{3} - \frac{\sin^5(3x)}{5}\right) + C}$

③ $\int \sin^2(2x)\cdot\cos^2(2x)\,dx$

both powers are even so...

$= \int \frac{1}{2}\left(1 - \cos(4x)\right)\cdot\frac{1}{2}\left(1 + \cos(4x)\right) dx$

$= \frac{1}{4}\int 1 - \cos^2(4x)\,dx = \frac{1}{4}\int 1\,dx - \frac{1}{4}\int \cos^2(4x)\,dx$

even power use which trig. identity?

$$= \frac{1}{4}x - \frac{1}{4}\int \frac{1}{2}\left(1 + \cos(8x)\right) dx$$

$$= \boxed{\frac{1}{4}x - \frac{1}{4}\cdot\frac{1}{2}\left(x + \frac{1}{8}\sin(8x)\right) + C}$$

4. $$\int \cos^3(7x)\cdot \sin^{1/2}(7x)\,dx$$

$$= \int \cos^2(7x)\cdot \sin^{1/2}(7x)\,\cos^1(7x)\,dx$$

$u = \sin(7x)$

$du = 7\cos(7x)dx$

$\frac{1}{7}du = \cos(7x)dx$

$$= \int \left(1 - \sin^2(7x)\right)\cdot \sin^{1/2}(7x)\,\cos(7x)\,dx$$

$$= \int (1 - u^2)\cdot u^{1/2}\cdot \frac{1}{7}du$$

$$= \frac{1}{7}\int u^{1/2} - u^{5/2}\,du = \frac{1}{7}\left(\frac{u^{3/2}}{3/2} - \frac{u^{7/2}}{7/2}\right) + C$$

$$= \boxed{\frac{1}{7}\left(\frac{\sin^{3/2}(7x)}{3/2} - \frac{\sin^{7/2}(7x)}{7/2}\right) + C}$$

5. $\int \cos^{-3}(x) \cdot \sin^3(x)\, dx$

$= \int \cos^{-3}(x) \cdot \sin^2(x) \cdot \sin^1(x)\, dx$

$u = \cos(x)$
$du = -\sin(x)\, dx$

$= \int \cos^{-3}(x) \cdot (1 - \cos^2(x)) \sin(x)\, dx$

$= -\int u^{-3} (1 - u^2)\, du$

$= -\int u^{-3} - u^{-1}\, du = \int -u^{-3} + \frac{1}{u}\, du$

$= -\frac{u^{-2}}{-2} + \ln|u| + C = \boxed{\frac{\cos^2(x)}{2} + \ln|\cos(x)| + C}$

TRIG SUBSTITUTION

6. $\displaystyle\int \frac{1}{\sqrt{4-x^2}}\,dx$

$\cancel{x = a\sin(\theta)}$ IGNORE

$x = 2\sin(\theta)$

$dx = 2\cos(\theta)\,d\theta$

$$= \int \frac{2\cos(\theta)\,d\theta}{\sqrt{4-(2\sin\theta)^2}} = \int \frac{2\cos\theta\,d\theta}{\sqrt{4-4\sin^2\theta}}$$

$$= \int \frac{2\cos\theta\,d\theta}{\sqrt{4(1-\sin^2\theta)}} = 2\int \frac{\cos(\theta)\,d\theta}{\sqrt{4(\cos^2\theta)}}$$

$$= \cancel{2}\int \frac{\cancel{\cos}\theta}{\cancel{2}\cancel{\cos}\theta}\,d\theta = \int 1\,d\theta = \theta + C$$

$$= \boxed{\sin^{-1}\left(\frac{x}{2}\right) + C}$$

ORIGINAL SUB

$x = 2\sin\theta$

$\frac{x}{2} = \sin\theta$

$\sin^{-1}\left(\frac{x}{2}\right) = \theta$

(7.) $\int \frac{1}{x\sqrt{9-x^2}}\,dx$

$x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

$$= \int \frac{\cancel{3}\cos\theta\,d\theta}{\cancel{3}\sin\theta\sqrt{9-(3\sin\theta)^2}}$$

$$= \int \frac{\cos\theta\,d\theta}{\sin\theta\sqrt{9-9\sin^2\theta}} = \int \frac{\cos\theta}{\sin\theta\sqrt{9(1-\sin^2\theta)}}\,d\theta$$

$$= \int \frac{\cos(\theta)\,d\theta}{\sin\theta\sqrt{9\cos^2\theta}} = \int \frac{\cos(\theta)\,d\theta}{\sin\theta\cdot 3\cdot\cos\theta}$$

$$= \frac{1}{3}\int \frac{1}{\sin\theta}\,d\theta = \frac{1}{3}\int \csc(x)\,dx$$

from new rule 2

$$= \frac{-1}{3}\ln|\csc(x) + \cot(x)| + C$$

from triangle below

$$= -\frac{1}{3}\ln\left|\frac{3}{x} + \frac{\sqrt{9-x^2}}{x}\right| + C$$

New Rule

① $\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$

AND

② $\int \csc(x)\,dx = -\ln|\csc(x) + \cot(x)| + C$

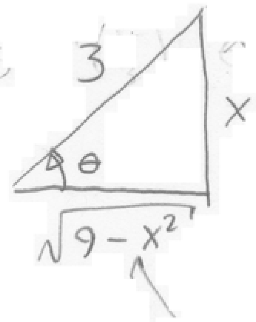
ORIGINAL SUBSTITUTION

$x = 3\sin\theta$

$\frac{x}{3} = \sin\theta$

$\to \csc(x) = \frac{3}{x}$

$\cot(x) = \frac{\sqrt{9-x^2}}{x}$



$A^2 + B^2 = C^2$

$A^2 + x^2 = 3^2$

$A^2 = 3^2 - x^2$

$A = \sqrt{9-x^2}$

8. $\int \frac{1}{\sqrt{x^2+7}}\,dx$ $\qquad x = \sqrt{7}\tan\theta$

$dx = \sqrt{7}\sec^2\theta$

$$= \int \frac{\sqrt{7}\sec^2(x)\,dx}{\sqrt{(\sqrt{7}\tan(x))^2+7}}$$

$$= \int \frac{\sqrt{7}\sec^2(x)\,dx}{\sqrt{7\tan^2(x)+7}} = \int \frac{\sqrt{7}\sec^2(x)\,dx}{\sqrt{7(\tan^2(x)+1)}}$$

$$= \int \frac{\sqrt{7}\sec^2(\theta)}{\sqrt{7\sec^2(\theta)}}\,d\theta = \int \frac{\cancel{\sqrt{7}}\sec^2\theta}{\cancel{\sqrt{7}}\sec\theta}\,d\theta$$

$$= \int \sec(\theta)\,d\theta = \ln|\sec(\theta) + \tan(\theta)| + C$$

New Rule (# 1) $\qquad$ FROM TRIANGLE

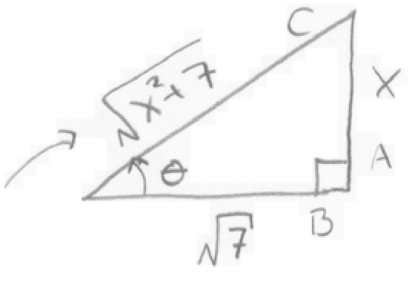
$$= \ln\left| \frac{\sqrt{x^2+7}}{\sqrt{7}} + \frac{x}{\sqrt{7}} \right| + C$$

ORIGINAL SUB.

$x = \sqrt{7}\tan\theta$

$\frac{x}{\sqrt{7}} = \tan\theta$

$$\sec(\theta) = \frac{1}{\cos\theta} = \frac{1}{\frac{A}{H}} = \frac{1}{\frac{\sqrt{7}}{\sqrt{x^2+7}}} = \frac{\sqrt{x^2+7}}{\sqrt{7}}$$



$A^2 + B^2 = C^2$

$x^2 + (\sqrt{7})^2 = C^2$

$$\boxed{\sqrt{x^2+7} = C}$$

(9.) $\int x^3\sqrt{5-x^2}\,dx$

$x=\sqrt{5}\sin\theta$

$dx=\sqrt{5}\cos\theta\,d\theta$

$$=\int(\sqrt{5}\sin\theta)^3\sqrt{5-(\sqrt{5}\sin\theta)^2}\;\sqrt{5}\cos\theta\,d\theta$$

$$=\int(\sqrt{5})^3\sin^3\theta\sqrt{5-5\sin^2\theta}\cdot\sqrt{5}\cos\theta\,d\theta$$

$$=(\sqrt{5})^3\cdot\sqrt{5}\cdot\int\sin^3\theta\sqrt{5(1-\sin^2\theta)}\cdot\cos\theta\,d\theta$$

$$=25\int\sin^3\theta\sqrt{5\cos^2\theta}\;\cos\theta\,d\theta$$

$$=25\sqrt{5}\int\sin^3\theta\cos^2\theta\,d\theta$$ ← NOW IT IS AN AT LEAST ONE ODD TRIG INT.

$$=25\sqrt{5}\int\sin^2\theta\cos^2\theta\sin^1\theta\,d\theta$$

$$=25\sqrt{5}\int(1-\cos^2\theta)\cos^2\theta\sin\theta\,d\theta$$

$u=\cos\theta$

$du=-\sin\theta\,d\theta$

$$=-25\sqrt{5}\int(1-u^2)\,u^2\,du$$

$$=-25\sqrt{5}\int u^2-u^4\,du=-25\sqrt{5}\left[\frac{u^3}{3}-\frac{u^5}{5}\right]+C$$

$$=-25\sqrt{5}\left[\frac{1}{3}\cos^3\theta-\frac{1}{5}\cos^5\theta\right]+C$$

$$=\boxed{-25\sqrt{5}\left[\frac{1}{3}\left(\frac{\sqrt{5-x^2}}{\sqrt{5}}\right)^3-\frac{1}{5}\left(\frac{\sqrt{5-x^2}}{\sqrt{5}}\right)^5\right]+C}$$

QUICK TRIANGLE

$\frac{x}{\sqrt{5}}=\sin\theta$