Name:____

MA 2320 Quiz 1 - Answers to question 3

1. Compute the Riemann sum approximation of $\int_1^3 2x^2+1$ with $n=3$.
   First we calculate Δx and x_k.
   $\Delta x = \frac{b-a}{n} = \frac{3-1}{3} = \frac{2}{3}$
   $x_k = a + k\Delta x = 1 + k\frac{2}{3}$

   Now we use these formulas to find each x_k.
   So $x_1 = 1 + (1)\frac{2}{3} = \frac{5}{3}$, $x_2 = \frac{7}{3}$ and $x_3 = 3$.
   $A_1 = f(x_1)\Delta X = f(\frac{5}{3})\frac{2}{3} = (2(\frac{5}{3})^2+1)\frac{2}{3} = \frac{118}{27}$
   $A_2 = f(x_2)\Delta X = f(\frac{7}{3})\frac{2}{3} = (2(\frac{7}{3})^2+1)\frac{2}{3} = \frac{214}{27}$
   $A_3 = f(x_3)\Delta X = \frac{342}{27}$

   So the approximation $= f(x_1)\Delta X + f(x_2)\Delta X + f(x_3)\Delta X = \frac{118}{27} + \frac{214}{27} + \frac{342}{27} = \frac{674}{3}$

2. Compute $\int_1^3 2x^2+1$ by using the definition of the integral.
   $\Delta x = \frac{b-a}{n} = \frac{3-1}{n} = \frac{2}{n}$
   $x_k = a + k\Delta x = 1 + k\frac{2}{n}$
   So $f(x_k) = 2(1+k\frac{2}{n})^2 + 1 = 2(1 + \frac{4k}{n} + \frac{4k^2}{n^2}) + 1 = 3 + \frac{8k}{n} + \frac{8k^2}{n^2}$.
   Thus

$$\begin{aligned}
\int_1^3 2x^2+1dx &= \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x \\
&= \lim_{n\to\infty}\sum_{k=1}^{n}(3 + \frac{8k}{n} + \frac{8k^2}{n^2})\frac{2}{n} \\
&= \lim_{n\to\infty}\sum_{k=1}^{n}\frac{6}{n} + \frac{16k}{n^2} + \frac{16k^2}{n^3} \\
&= \lim_{n\to\infty}\sum_{k=1}^{n}\frac{6}{n} + \sum_{k=1}^{n}\frac{16k}{n^2} + \sum_{k=1}^{n}\frac{16k^2}{n^3} \\
&= \lim_{n\to\infty}\sum_{k=1}^{n}\frac{6}{n} + \frac{16}{n^2}\sum_{k=1}^{n}k + \frac{16k^2}{n^3}\sum_{k=1}^{n}k^2 \\
&= \lim_{n\to\infty}\frac{6}{n}(n) + \frac{16}{n^2}\frac{n(n+1)}{2} + \frac{16k^2}{n^3}\frac{n(n+1)(2n+1)}{6} \\
&= 6 + \frac{16}{2} + \frac{16}{3} = \frac{58}{3}
\end{aligned}$$