Math 7500 - Test 2

Name:_____

1 Student problems - Show all work

1. Deena - (law of sines/law of cosines)
   Triangle ABC has sides with lengths $a = 5$, $b = 7$ and $c = 3$. Find the angles A, B and C. Orient the triangle with base AB (so $AB = c = 3$), find the height of the triangle and its area.

2. Michael - ($A = A_0 e^{rt}$)
   I have a bank account and I get an annual interest rate of 2 % . When will my bank account double in value?

3. Christine - (sine, cosine modeling)
   A river flows over a cliff into a water wheel to power the mill. The top of the water wheel is at ground level (height of zero feet and the water wheel has a diameter of 75 feet (so the bottom of the water wheel is 75 feet below ground level). The water pushes the water wheel around at 3 revolutions per minute. Model the height of a particular spot on the water wheel with sine or cosine (mention units).

4. Lauren - Ode to a Grecian Urn
   Assume we have a bin with colored balls: 3 red balls, 4 purple balls and 5 green balls. We draw two balls. Compute the

   (a) probability that the two balls are the same color and

   (b) probability that the balls are purple given that the two balls are the same color

5. Krystal - Completing the square
   Complete the square of $0 = 3x^2 + 2x - 2$ to solve.

2 Regular CST questions - From sample + 6 new questions

6. Use the information below to answer the question that follows. Let

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \text{ and } B = \begin{bmatrix} a & b & c \\ d+2a & e+2b & f+2c \\ g & h & i \end{bmatrix}$$

Matrix B is composed of the elements from matrix A, as shown above. Which statement describes the relationship between the determinants of the two matrices?

(a) det(B) = det(A)

(b) det(B) = -det(A)

(c) det(B) = 2 det(A)

(d) det(B) = 1/2 det(A)

7. Use the graph below to answer the question that follows.

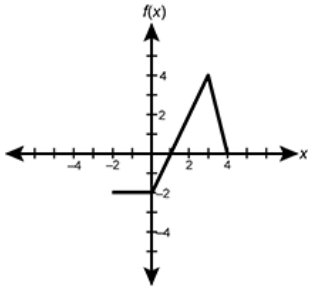

Figure 1:

The graph of a function f(x) is shown above. What is the value of $\int_0^4 2f(x) + 1dx$?

(a) 11

(b) 14

(c) -6

(d) 12

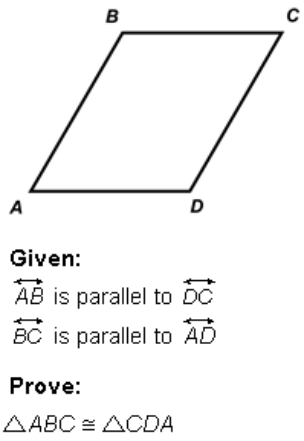

Figure 2:

8. A student is assigned the following proof.

   The student reasons as follows:

   Draw a line between point A and point C. Then, angle BCA is congruent to angle DAC. Also note that angle BAC is congruent to angle DCA. Now, AC $\equiv$ co AC so the two triangles are congruent by angle-side-angle.

   Which statement best justifies the second and third sentences in the student's response?

   (a) If two lines are cut by a transversal and same-side interior angles are congruent, then the lines are parallel.

   (b) If two lines are cut by a transversal and alternate interior angles are congruent, then the lines are parallel.

   (c) If two parallel lines are cut by a transversal, then same-side interior angles are congruent.

   (d) If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

9. A bottling company uses a machine to fill juice bottles. The quantity of juice that goes into each bottle is normally distributed, with a mean of 20 ounces and a standard deviation of 0.5 ounces. Approximately what percentage of the bottles receives less than 19.25 ounces?

   (a) 36.2%

(b) 6.7%
(c) 9.5%
(d) 11.3%

10. Which of the following is always composite if $x, y \in \{1, 2, 3, \ldots\}$

(a) $x + y$
(b) $x(y + 2)$
(c) $(x + 1)(y + 2)$
(d) $5x + 3y$

11. Solve for x: $|2x + 1| > 3$

(a) $x = 1, -2$
(b) $(-2, 1)$
(c) $(-\infty, -2) \cup (1, \infty)$
(d) $(1, \infty)$

12. The function $2x + y = 4$ has x-intercept:

(a) 1
(b) 2
(c) 3
(d) 4

13. A circle with endpoints $(-3, 0)$ and $(3, 2)$ has equation

(a) $x^2 + y^2 - 2x = 10$
(b) $x^2 + y^2 - 2y = 9$
(c) $x^2 + y^2 = 10$
(d) $x^2 + y^2 = 9$

14. A light year is the distance light can travel in a year. Light travels at a speed of 299,792,458 meters / sec. How many miles is a light year. Hint: 1 meter = 3.281 feet and 5280 feet in one mile. It is up to you to remember how many seconds in an hour.

(a) $5.88 * 10^{12}$

(b) $3.76 * 10^{10}$

(c) $6.92 * 10^{13}$

(d) $8.78 * 10^{21}$

15. Given a a cylindrical tank 10 feet deep and with a circular base with radius of 4 feet. There is a 10 inch pipe running up and down through the center of the tank. The tank is filled half way with water (no water is in the pipe), how much water approximately is in the tank? (Hint: 1 cubic foot is 7.48052 gallons.)

    (a) 1800 gallons

    (b) 1900 gallons

    (c) 240 gallons

    (d) 578 gallons

16. Which of the following is not equal* to $\sin(x)$

    (a) $\sqrt{1-\cos^2(x)}$

    (b) $\frac{\sin(2x)}{\cos(x)}$

    (c) $\frac{1+\cos(2x)}{2\sin(x)}$

    (d) $\frac{\tan(x)}{\sec(x)}$

    * where $0 \leq \theta \leq \pi$.

17. What is the domain of the function: $f(x) = \frac{3x-6}{x^2-25}$

    (a) $x \neq 2$

    (b) $x \neq 5, -5$

    (c) $x \neq 2, -2$

    (d) $x \neq 5$

18. Which of the following is a factor of $6 + 48m^3$.

    (a) $1 + 2m$

    (b) $1 - 8m$

    (c) $1 + m - 2m$

    (d) $1 - m + 2m$

19. Which of the following is equivalent to $\frac{(x^{1/b})^a}{x^a}$

    (a) $x^{1/b}$
    (b) $x^{a-1/b}$
    (c) $x^{(a-1)/b}$
    (d) $x^{(a-ab)/b}$

20. A parabola opening toward the negative x-axis with vertex $(1, 2)$ has equation.

    (a) $(y-1) = -(x-2)^2$
    (b) $(x-1) = (y-2)^2$
    (c) $(x-1)^2 = -(y-2)$
    (d) $(y-1)^2 = -(x-2)^2$

21. How does the function $f(x) = x^3 + x^2 - x + 4$ behave over the interval $(1, 3)$.

    (a) increasing
    (b) decreasing
    (c) increasing then decreasing
    (d) neither increasing nor decreasing

22. Find the area under the function $y = x^2 + 4$ from $x = -1$ to $x = 1$.

    (a) 10
    (b) 26/3
    (c) -10
    (d) 20

23. Compute the median for $\{12, 13, 17, 19, 19\}$

    (a) 15
    (b) 16
    (c) 17
    (d) 16.5

24. If the correlation between two variables is zero then what must be true

(a) negative linear

(b) there is no relation between the two variables

(c) quadratic

(d) the relationship is not linear

25. A fair coin is flipped five times. Compute the probability that the third head occurs on the fifth flip.

(a) $\frac{6}{32}$

(b) $\frac{1}{32}$

(c) $\frac{10}{32}$

(d) $\frac{1}{16}$

26. A fair die is tossed five times. Compute the probability that the exactly four out of the five rolls are strictly greater than 2.

(a) $\frac{80}{243}$

(b) $\frac{80}{6^5}$

(c) $\frac{10}{243}$

(d) $\frac{1}{6^5}$

27. Compute the product AB

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ 3 & 0 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \\ 1 & 2 \end{bmatrix}$$

(a) $\begin{bmatrix} -1 & -1 \\ 0 & 5 \\ -1 & 7 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 0 \\ -1 & 5 \\ -1 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 & -1 \\ -1 & 5 & 7 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & -1 & -1 \\ 0 & 5 & 7 \end{bmatrix}$

(e) cannot be computed

28. Compute the product BA

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ 3 & 0 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \\ 1 & 2 \end{bmatrix}$$

(a) $\begin{bmatrix} -1 & -1 \\ 0 & 5 \\ -1 & 7 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 0 & -1 \\ -1 & 5 & 7 \end{bmatrix}$

(c) cannot be computed

29. Compute the product $A^{-}1$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ 3 & 0 & 2 \end{bmatrix}$$

(a) entry in first row and first column of 1

(b) entry in first row and first column of $\frac{2}{3}$

(c) entry in first row and first column of $\frac{-1}{3}$

(d) entry in first row and first column of -1

(e) there is no inverse

30. Which is the definition of the following sequence

$$\{2, 3, 6, 18, \ldots\}$$

(a) $N_i = \frac{3}{2}N_{i-1}$

(b) $N_i = N_{i-1}N_{i-2}$

(c) $N_i = N_{i-1} + N_{i-2} + 1$

(d) $N_i = N_{i-1} + N_{i-2}$

(e) none of the above

31. Compute the sum of the sequence

$$\{3, 5, 7, 8, 11, 13, \ldots, 101\}$$

(a) 3160

(b) 5150

(c) 2600

(d) 4000

(e) none of the above

32. Compute the sum of the sequence

$$\{2, -1, \frac{1}{2}, \frac{-1}{4}, \frac{1}{8}, \ldots, \frac{1}{2048}\}$$

(a) $\frac{2^{13}+1}{2^{13}}$

(b) $\frac{2^{13}+1}{3 \cdot 2^{11}}$

(c) $\frac{1}{2^{13}}$

(d) none of the above

3 Optimization

Use the diagram and the information below to complete the exercise that follows.

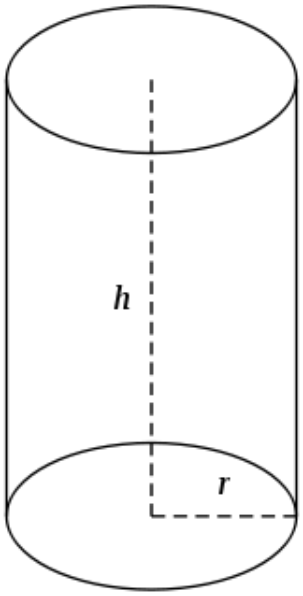

Figure 3:

The diagram shows a right cylinder with a radius r and height h. A beverage company wants to make a right cylindrical can that holds 900 cm^3 of juice. Assume the thickness of the material used to make the can is negligible.

- Use your knowledge of volume and surface area to derive a function A(r) that represents the surface area of the can in terms of the radius, r, of its base;
- use your calculator to produce a graph of A(r) that shows the intercepts, extrema, and asymptotic behavior of the function over the set of real numbers, sketch the graph you obtained, and identify the window you used;
- state any restriction on the domain of the function so that it represents the physical model of the can; and

- use your calculator to find the dimensions of the can to the nearest tenth of a centimeter that will minimize the quantity of material needed to manufacture the can.

Be sure to show your work and explain the steps you used to obtain your answers.