Math 7500 - New Version Practice CST - 2014

This documnet is a listing of the new Math CST syllabus and a list of questions that are my guess as to what a question will be.

0 CST Guidelines

	Items Approximate	Percentage of Test Score
0001 Number and Quantity	9	8%
0002 Algebra	23	20%
0003 Functions	19	17%
0004 Calculus	11	10%
0005 Geometry and Measurement	17	15%
0006 Statistics and Probability	11	10%
0007 Pedagogical Content Knowledge	1	20%

0.1 COMPETENCY 0001: NUMBER AND QUANTITY

Performance Expectations The New York State Mathematics teacher understands and extends concepts of number and quantity, from the properties of arithmetic operations involving real numbers through the properties of operations involving vector and matrix representations and complex numbers.

Performance Indicators

(a) applies and extends understanding of arithmetic to the rational numbers

(b) applies properties of rational numbers to solve real-world and mathematical problems involving the four operations with rational numbers

(c) applies and extends understanding of integer exponents to include rational exponents and rewrites expressions involving radicals and rational numbers

(d) reasons quantitatively and uses appropriate units to solve problems

(e) demonstrates understanding of the properties of real numbers and applies real numbers to model and solve multistep problems

(f) performs arithmetic operations with complex numbers

(g) represents complex numbers and their operations in the complex plane, using both rectangular and polar forms

(h) uses complex numbers to factor and solve quadratic equations and applies the fundamental theorem of algebra

(i) models and solves problems with vector quantities, including problems involving velocity and other quantities represented by vectors

(j) performs arithmetic operations (e.g., addition, subtraction, scalar multiplication) on vectors and represents vectors, their magnitudes, and vector operations symbolically and graphically

(k) demonstrates understanding of the properties of matrices, performs operations on matrices, and uses matrices in applications

(l) demonstrates knowledge of abstract algebra (e.g., groups, rings, fields, vector spaces)

0.2 COMPETENCY 0002: ALGEBRA

Performance Expectations The New York State Mathematics teacher understands the use of numbers, symbols, operations, and conventions of notation that allow the creation, interpretation, and manipulation of algebraic expressions and equations, and uses them to model and solve mathematical and real-world problems.

Performance Indicators

(a) uses properties of operations to generate equivalent expressions and solves real- life and mathematical problems using numerical and algebraic expressions and equations

(b) analyzes rates and proportional relationships and uses them to solve real-world and mathematical problems

(c) analyzes connections between proportional relationships, lines, and linear equations

(d) interprets the structure of expressions and rewrites expressions in equivalent forms (e.g., factoring, completing the square in a quadratic expression, transforming exponential expressions and equations, finding the sum of a finite geometric series)

(e) performs arithmetic operations on polynomials, simplifies polynomial expressions using identities, and expands binomials

(f) demonstrates understanding of the relationship between zeros and factors of polynomials and extends polynomial identities to the complex numbers

(g) rewrites and manipulates rational expressions

(h) creates equations and inequalities in one, two, or more variables to describe numbers or relationships (e.g., linear, quadratic, exponential), including situations involving constraints, and interprets the viability of options in modeling contexts

(i) understands solving equations and equalities as a process of reasoning and explains the reasoning, including situations when extraneous solutions may arise

(j) solves linear equations and inequalities and quadratic equations in one variable

(k) solves systems of linear and quadratic equations using a variety of methods (e.g., algebraic, graphic, matrix)

(l) represents and solves linear and nonlinear equations and inequalities graphically

0.3 COMPETENCY 0003: FUNCTIONS

Performance Expectations The New York State Mathematics teacher understands that functions are descriptions, often in the form of algebraic expressions, of situations in which one quantity depends on another, and that functions have many applications modeling nature and human society.

Performance Indicators

(a) demonstrates understanding of the concept of a function and the use of function notation, including sequences and recursive functions

(b) interprets functions that arise in applications in terms of context and interprets key features (e.g., domain, intercepts, rate of change, end behavior, periodicity) of functional relationships presented in written descriptions, symbolic expressions, tables, or graphs

(c) uses different representations to analyze functions (e.g., linear, quadratic, radical, rational, piecewise, absolute value, exponential, logarithmic)

(d) builds functions that model relationships between two quantities using a variety of methods (e.g., explicit expressions, recursive processes, arithmetic combination of functions, composition of functions)

(e) analyzes arithmetic and geometric sequences both recursively and with an explicit formula, translates between the two forms, and uses them to model situations

(f) builds new functions from existing functions and analyzes their graphs (e.g., analyzes the effect of replacing f(x) with f(x) + k, kf(x), f(kx) f(x + k)), finds and analyzes inverse functions, and identifies even and odd functions

(g) compares and contrasts linear, quadratic, and exponential functions, and uses them to model and solve problems

(h) solves problems involving logarithmic and exponential functions

(i) analyzes trigonometric functions using the unit circle

(j) models periodic phenomena with trigonometric functions

(k) proves and applies trigonometric identities

(l) solves trigonometric equations

0.4 COMPETENCY 0004: CALCULUS

Performance Expectations The New York State Mathematics teacher understands the fundamental concepts of calculus and how techniques of calculus are essential in the modeling and solving of both mathematical and real-world problems.

Performance Indicators

(a) analyzes the concept of limits and applies it to interpret the properties of functions (e.g., continuity, asymptotes)

(b) interprets derivatives and definite integrals as limits (e.g., difference quotients, slope, Riemann sums, area)

(c) applies the fundamental theorem of calculus

(d) applies techniques of differentiation and integration (e.g., product rule, chain rule, u-substitution)

(e) applies properties of derivatives to analyze the graphs of functions

(f) demonstrates knowledge of power series

(g) uses derivatives to model and solve mathematical and real-world problems (e.g., rates of change, related rates, optimization)

(h) uses integration to model and solve mathematical and real-world problems (e.g., work, applications of antiderivatives)

(i) models and solves problems involving first order differential equations (e.g., separation of variables, initial value problems)

0.5 COMPETENCY 0005: GEOMETRY AND MEASUREMENT

Performance Expectations The New York State Mathematics teacher understands the attributes and relationships of geometric objects in diverse contexts and applies the properties of measurement and dimension in modeling situations.

Performance Indicators

(a) understands the Pythagorean theorem and its converse (including proofs) and applies the theorem to solve problems in two and three dimensions and in the coordinate plane

(b) analyzes and applies properties of rotations, reflections, and translations in the plane and demonstrates understanding of congruence in terms of rigid motions

(c) proves and applies theorems about lines and angles, triangles, and parallelograms

(d) proves and analyzes geometric constructions

(e) demonstrates understanding of similarity in terms of similarity transformations and proves theorems involving similarity

(f) applies right triangle trigonometry to solve problems, and applies trigonometry to general triangles (e.g., law of sines, law of cosines)

(g) applies theorems about circles and solves measurement problems involving circles (e.g., arc lengths, areas of sectors)

(h) translates between geometric descriptions and equations for conic sections (e.g., circles, parabolas, ellipses, hyperbolas)

(i) uses coordinates to prove simple geometric theorems algebraically (e.g., slope criteria for parallel and perpendicular lines, properties of polygons)

(j) demonstrates understanding of area and volume formulas and Cavalieri's principle, and uses them to model and solve problems

(k) identifies relationships between two-dimensional and three-dimensional objects

(l) applies geometric concepts in modeling situations (e.g., using shapes to describe objects, applying concepts of density based on area and volume)

(m) demonstrates knowledge of non-Euclidean geometry

0.6 COMPETENCY 0006: STATISTICS AND PROBABILITY

Performance Expectations The New York State Mathematics teacher understands that information contained in data is often obscured by variability and uses statistical tools and knowledge of probability to make informed decisions that allow for this variability.

Performance Indicators

(a) summarizes and represents data on a single count or measurement variable (e.g., using number lines, dot plots, histograms, and box plots)

(b) summarizes and represents data on two categorical and quantitative variables (e.g., using two-way frequency tables and scatter plots)

(c) interprets one- and two-variable data presented in a variety of formats (e.g., analyzing data plots in terms of mean, median, interquartile range, standard deviation, and outliers; interpreting shape, center, and spread of data sets; using the mean and standard deviation to fit data to a normal distribution and analyze the fit; analyzing trends; fitting functions to data; plotting and analyzing residuals)

(d) interprets correlation coefficients for linear models and distinguishes between correlation and causation

(e) demonstrates understanding of random processes, random variables, and probability distributions (e.g., normal, binomial, uniform distributions)

(f) evaluates statistical experiments (e.g., making inferences about population parameters from a single random sample)

(g) recognizes the purposes of and differences between sample surveys, experiments, and observational studies and makes inferences and justifies conclusions from them

(h) demonstrates understanding of independence and conditional probability and uses them to interpret data

(i) uses the rules of probability (e.g., addition rule, multiplication rule) and/or permutations and combinations to compute probabilities of compound events in a uniform probability model

(j) calculates expected values and uses them to solve problems

(k) uses probabilities to evaluate outcomes of decisions

0.7 COMPETENCY 0007: PEDAGOGICAL CONTENT KNOWLEDGE

Performance Expectations The New York State Mathematics teacher effectively applies pedagogical content knowledge across multiple content domains to design instruction to help students achieve a specific learning goal. The teacher analyzes student understanding and identifies potential and apparent student difficulties. The teacher applies knowledge of how students learn to develop an effective instructional strategy that includes multiple ways of representing mathematical concepts and procedures that will facilitate development of students' skills and their achievement of the desired learning goal.

Performance Indicators

(a) identifies the skills and conceptual understanding necessary for students to achieve a specific new learning goal

(b) demonstrates knowledge of methods for assessing student readiness for a specific new learning goal

(c) demonstrates knowledge of ways to connect students' prior learning to the new learning goal

(d) promotes coherence by connecting learning across the mathematical domains

(e) describes an appropriate and effective instructional strategy that includes multiple representations of essential/difficult concepts

(f) demonstrates knowledge of methods for assessing students' progress during the lesson toward achieving the learning goal

1 Competency 0001: Number and Quantity

(a) applies and extends understanding of arithmetic to the rational numbers

Problem 1.1. *Solve:*

- $\left(\frac{8}{3}+\frac{7}{2}\right)\cdot\frac{2}{5}$
- $\frac{\frac{3}{4}-2}{\frac{5}{6}+1}$

(b) applies properties of rational numbers to solve real-world and mathematical problems involving the four operations with rational numbers

Problem 1.2.
- *Adam's paving company can pave 400 feet of one lane road per day. How many feet per day can adam pave a three lane road.*
- *How to pick your raise. Smith and Jones were each hired at $10,000 per year and offered a choice of two different raise packages:*
  (1) every six months you get a $500 raise on your six month salary, or
  (2) every year you get a $1600 dollar raise on your 1one year salary.

  Smith chose option one, Jones option 2. Who made more money after the three year contract? Explain.

(c) applies and extends understanding of integer exponents to include rational exponents and rewrites expressions involving radicals and rational numbers

Problem 1.3. *Simplify the following:*

- $\left(\frac{x^{1/2}y^2}{\sqrt{x^3y}}\right)^3$
- $(2a^{1/2}b^2)^2+(3a^{1/3}b^{1.5})^3+(4a^{1/4}b^1)^4$
- $\sqrt{32x^3y^4}+\sqrt[3]{32x^3y^4}$
- $\frac{1}{2-\sqrt{2}}+\frac{\sqrt{3}}{2-\sqrt{3}}$
- $\frac{1}{2-\sqrt[3]{2}}$ *(rationalize)*

(d) reasons quantitatively and uses appropriate units to solve problems

Problem 1.4. • *We watch a stream and count the number of fish going by and counted 3 fish swam by in one hour. But over the winter the number of fish swimming is lower and we watched for a full 5 hours and only saw 2 fish. Assume the the fish swim past at the first rate for 270 days per year and the winter rate for 95 days of the year. How many fish per year?*

- *The following units are mass (m) in kg, velocity (v) meters per second, acceeration (a) meters per second, energy (e) kg meters squared per second squared. What are the units of*

$$\frac{mv^2}{ea}.$$

- *My space ship travels at the rate 0.2 c (c is the speed of light 300,000,000 meters/sec). I have a very fast space ship. But I was just pulled over by the Nassau County Police Department and they asked do I know how fast I am going. I said only 0.1 c and the said inniles per hours please. Help by converting 0.1 c to miles per hour. Hint 1.6 km = 1 mile.*

(e) demonstrates understanding of the properties of real numbers and applies real numbers to model and solve multistep problems

Problem 1.5. • *We need a spherical ballon to hold 100 cubic inches of volume. What should the radius be?*

- *My car gets 30 miles per gallon and I need some gas. If I go to the gas station A is 1 mile away and gas costs $3.00 per gallon of gas and gas station B is 7 miles away and gas costs $2.90 per gallon of gas. I am at a loss of what to do. On sation is closer, the other is cheaper. Assume I get $20.00 of gas. How much additional gas i in my tank chosing gas station A or B? Remember I need to go to the gas station and come back home.*

(f) performs arithmetic operations with complex numbers

Problem 1.6. *Solve*

- $(2 - i)(i + 4)$
- $\frac{1}{2+i} - \frac{3+i}{3-2i}$ *(rationalize)*

(g) represents complex numbers and their operations in the complex plane, using both rectangular and polar forms

Problem 1.7. *Solve*

- *write* $2 - i$ *in polar.*
- *write* e^{2-2i} *in rectangular.*
- *write* $e^1(\cos(\pi/3) + i\sin(\pi/3))$ *in rectangular.*
- *simplify* $e^{2-2i} * e^1(\cos(\pi/3) + i\sin(\pi/3))$
- *simplify* $e^{2\bar{-}2i}$ *(the conjugate)*

(h) uses complex numbers to factor and solve quadratic equations and applies the fundamental theorem of algebra

Problem 1.8.

- *Factor* $x^2 + 1$ *using complex numbers.*
- *Factor* $x^3 + 1$ *using complex numbers.*
- *What are the roots of* $x^2 + 1$ *using complex numbers.*
- *What are the roots of* $x^3 + 1$ *using complex numbers.*
- *How many roots (counting multiplicity) does* $x^1 1 + 1$ *have (using complex numbers).*

(i) models and solves problems with vector quantities, including problems involving velocity and other quantities represented by vectors

Problem 1.9.

- *A stranded barge is being pulled by two tug boats. The first is pulling at 30 mph and the second is pulling at 25 mph. The angle between the tug boats tow ropes is* $45°$. *What is the speed the barge is moving at and at what angle relative to the tow ropes.*
- *A ball is thrown in the air at time t=0 seconds with position vector in meters*

  $$\mathbf{r}(t)\langle 3t, -4.9t^2 + 4t\rangle$$

  How long is the ball in the air? How far does the ball travel before it hits the ground? What is the speed of the ball intitially? At what angle is the ball thrown into the air?
- *Compute the the vectors* $\mathbf{F_d}$ *and* $\mathbf{F_n}$ *if the weight is 100 lbs (see figure 1.9).*

(j) performs arithmetic operations (e.g., addition, subtraction, scalar multiplication) on vectors and represents vectors, their magnitudes, and vector operations symbolically and graphically

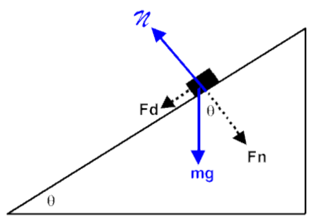

Figure 1:

Problem 1.10. • *What is the vector from the point* $P(1,2,3)$ *to* $Q(-4,2,3)$.

- *Compute the addition of the vectors below and sketch the two vectors and their sum on an xy-graph.*

$$\langle 1,2\rangle + \langle -2,3\rangle$$

- *find the angle between the two vectors* $\langle 1,2\rangle$ *and* $\langle -2,3\rangle$.
- *Solve for* a *and* b*:*

$$-2\langle 1,2\rangle + 4\langle a,3\rangle = \langle 2,b\rangle$$

- *Compute the following.*
  - $\langle 1,2,3\rangle \cdot \langle 0,1,-1\rangle$
  - $\langle 1,2,3\rangle \times \langle 0,1,-1\rangle$
  - *Find the angle between* $\langle 1,2,3\rangle$ *and* $\langle 0,1,-1\rangle$
  - *Find the area of the parallelogram formed by the two vectors* $\langle 1,2,3\rangle$ *and* $\langle 0,1,-1\rangle$
  - $\|\langle 1,2,3\rangle\|\|\langle 0,1,-1\rangle\|$

(k) demonstrates understanding of the properties of matrices, performs operations on matrices, and uses matrices in applications

Problem 1.11. • *compute the determinant of* A*,* B*,* C *and* D*.*

$$A = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -1 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -1 & 3 \\ 1 & -1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} -1 & 3 \\ 2 & 3 \\ 2 & 1 \end{bmatrix}$$

- *compute the inverse of the above matrices. If not possible, why?*
- *What are the determinants of the inverse matrices you computed?*
- *compute the products AB, AD, DA, CD and DC. If not possible, why?*
- *what is the determinant of BC. Answer without computing the product BC.*
- *what are the three basic row operations and what effect do they have on the determinant?*
- *Solve the following using matrices and inverse matrices.*

$$\begin{aligned} 2x + y &= 2 \\ 3x + 5y &= 3 \end{aligned}$$

- *Solve the following using matrices and Gauss-Jordan elimination.*

$$\begin{aligned} 2x + y &= 2 \\ 3x + 5y + z &= 3 \\ x - 2z &= 1 \end{aligned}$$

(l) demonstrates knowledge of abstract algebra (e.g., groups, rings, fields, vector spaces)

Problem 1.12.

- *For the ring ($\mathbb{Z}$, +, *), what is the additive identity.*
- *For the group ($\mathbb{Q}^*$, *), what is the identity.*
- *Is the Group of invertible matrices associative? commutative?*
- *Identify the inverse of $\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$ for the following group*

$$\{1, \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}, i, -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}, -1, -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}, -i, \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\}, *$$

- *Which of the following are groups. If not, why?*
  - *($\mathbb{Z}$, +)*

- *($\mathbb{Z}$, *)*
- *($\mathbb{Q}$, *)*
- *($\mathbb{C}$, +)*
- *2x2 Matrices excluding the zero matrix over multiplication.*
- *2x2 Matrices excluding the zero matrix over addition.*

2 Competency 0002: Algebra

(a) uses properties of operations to generate equivalent expressions and solves real-life and mathematical problems using numerical and algebraic expressions and equations

(b) analyzes rates and proportional relationships and uses them to solve real-world and mathematical problems

Problem 2.1.

- *Going to visit Grandma for the weekened I drove the first 100 miles at 30 mph, and I drove the next 200 miles at 50 mph. How long did I drive? How far (many miles) was the trip?*
- *Going to visit Grandma for the weekened I drove the first hour at 30 mph, and I drove the next two hours at 50 mph. How long did I drive? How far (many miles) was the trip?*
- *Once again visiting Grandma for the weekened I drove the first hour at 30 mph. But I want to average 40 mph. How fast should I drive the remaining 200 miles of the trip?*
- *We have a conical tank of water with dimensions 100 feet tall and a radius of 20 feet at the top of the tank. Find a function of height that represents how much water is in the tank at a given height.*
- *We are draining a conical tank at a rate of 10 cubic feet of water per minute. The conical tank has dimensions 100 feet tall and has radius of 20 feet at the top of the tank. That tank has 80 feet of water initially (at t=0). Find an equation that represents how fast the tank is emptying (in minutes).*

(c) analyzes connections between proportional relationships, lines, and linear equations

Problem 2.2.

- *I have a conical tank of water that has colleced rain water. I will use it to fill my pool. The conical tank is 100 feet tall and has a 25 feet radius at the top of the tank. With the heavy rains the tank is filled up to 90 feet. How much must I drain the conical tank to fill my pool? My pool has dimensions width is 10 feet Length is 30 feet and the pool is 4 feet deep at the shallow end (at length= 0) and 8 feet at the deep end (at length = 30).*

(d) interprets the structure of expressions and rewrites expressions in equivalent forms (e.g., factoring, completing the square in a quadratic expression, transforming exponential expressions and equations, finding the sum of a finite geometric series)

Problem 2.3. • *Factor* $4x^3 - 64$

- *Complete the square to solve* $x^2 + 4x - 5 = 0$.
- *Use the quadratic equation to solve* $x^2 + 4x - 5 = 0$.
- *Use the quadratic equation to solve* $3z^6 - 3z^3 - 5 = 0$.
- *Find the sum* $3 + 6 + 12 + 24 + 48 + \cdots + 192$.
- *Find the sum* $8 - \frac{8}{3} + \frac{8}{9} - \frac{8}{27} + \frac{8}{81} \cdots - \frac{8}{3^11}$.
- *Solve for* x, $e^{3x} = 2$.
- *Solve for* x, $2 \cdot 4^{x/2} = 11$.
- *Solve for* x, $2 \cdot 4^{x^2+x} = 11$.
- *Solve for* x, $2\log_2(4x + 1) = -1$.

(e) performs arithmetic operations on polynomials, simplifies polynomial expressions using identities, and expands binomials

Problem 2.4. *Let* $f(x) = x^3 + 2x^2 - 1$ *and* $g(x) = x^4 + 2x - x$.

- *Simplify* $2g(x) - xf(x)$
- *Expand* $(2x - 1)^4$
- *Simplify* $(a + b)^3 - (a - b)^3$

(f) demonstrates understanding of the relationship between zeros and factors of polynomials and extends polynomial identities to the complex numbers

Problem 2.5. • *Factor* $x^3 - 1$. *Find all real zeroes.*

- *Factor* $x^3 - 1$. *Find all complex zeroes.*
- *Find a polynomial with zeroes 1,2,3 and has order 4.*
- *Factor* $5x^2 + 2x - 11$.
- *Find zeroes of* $f(x) = 5x^2 + 2x - 11$.

(g) rewrites and manipulates rational expressions

Problem 2.6. • *Simplify* $\dfrac{x+1}{2x-1} + \dfrac{1}{x+1}$

- *Simplify* $\dfrac{x+1}{2x-1} \div \dfrac{1}{x+1}$
- *Simplify* $\dfrac{x^2-1}{x^2+5x+6} \div \dfrac{x^2-4}{x^2+6x+9}$

- *Simplify* $\dfrac{x^2-1}{x^2+5x+6} \div \dfrac{x^2-4}{x^2+6x+9}$

(h) creates equations and inequalities in one, two, or more variables to describe numbers or relationships (e.g., linear, quadratic, exponential), including situations involving constraints, and interprets the viability of options in modeling contexts

Problem 2.7. • *Let n be the number of books sold. The books sell for \$7.00 each. Write an equation for revenue, R, as a function of n. Graph that function and graph the region that represents region larger than revenue.*

- *When we price the books higher we see a loss in demand (number sold, n). So at \$7.00 each we sell 100 books, and at \$8.00 we sell 88 books. What is the demand function, n, as a function of p, price.*
- *So using the previous problem We have a better revenue function. Compute and graph the new revenue function as a function of p (Recall revenue is $n * p$).*

(i) understands solving equations and equalities as a process of reasoning and explains the reasoning, including situations when extraneous solutions may arise

Problem 2.8. • *Solve* $x^2+2x-2>0$.

- *Solve* $\sqrt{x}=2x+2$
- *Solve* $\sqrt{x}>2x+2$
- *Let* $h(t) = -4.9t^2+4t+6$ *represent the height of a ball thrown in the air from the second story window of the NAB with height in metters and the time in seconds. Find when the ball hits the ground (that is when $h=0$).*

(j) solves linear equations and inequalities and quadratic equations in one variable

Problem 2.9. • *Solve the following simultaneous equations using substitution.*

$$\begin{aligned} 2x+y &= 2 \\ 3x+5y &= 3 \end{aligned}$$

- *Solve the following simultaneous equations using substitution.*

$$\begin{aligned} 2x + y &= 2 \\ 3x + 5y + z &= 3 \\ x - 2z &= 1 \end{aligned}$$

$$\begin{aligned} x + y + z &= 2 \\ 3x + 5y + z &= 3 \\ x + 3y - z &= 1 \end{aligned} \quad \textit{and} \quad \begin{aligned} x + y + z &= 2 \\ 3x + 5y + z &= 3 \\ x + 3y - z &= -1 \end{aligned}$$

- *Graph the solution to the following simultaneous inequalities.*

$$\begin{aligned} 2x + y &> 2 \\ 3x + 5y &\leq 3 \end{aligned}$$

(k) solves systems of linear and quadratic equations using a variety of methods (e.g., algebraic, graphic, matrix)

Problem 2.10.

- *Solve the systems graphically.*

$$\begin{array}{rrl} x & +y & = 2 \\ 2x & -y & = 2 \end{array} \quad \textit{and} \quad \begin{array}{rrl} x & +y & = 2 \\ 3x & +3y & = 2 \end{array}$$

- *Solve the systems with substitution.*

$$\begin{array}{rrl} 2x & +y & = 2 \\ 4x & -y & = 2 \end{array} \quad \textit{and} \quad \begin{array}{rrrl} x & +y & +z & = 2 \\ 3x & +3y & +z & = 2 \\ & -3y & +z & = 2 \end{array}$$

- *Solve the systems with elimination.*

$$\begin{array}{rrl} 2x & +y & = 2 \\ 4x & -y & = 2 \end{array} \quad \textit{and} \quad \begin{array}{rrrl} x & +y & +z & = 2 \\ 3x & +3y & +z & = 2 \\ & -3y & +z & = 2 \end{array}$$

- *Solve the systems with some matrix method, know both matrix methods and Gaussian elimination.*

$$\begin{array}{rrrl} x & +y & +z & = 2 \\ 3x & +3y & +z & = 2 \\ & -3y & +z & = 2 \end{array} \qquad \begin{array}{rrrl} x & +y & +z & = 2 \\ 3x & +3y & +z & = 2 \\ & -3y & +z & = 2 \end{array}$$

$$\begin{array}{rrrl} x & +y & +z & = 2 \\ 3x & +3y & +z & = 2 \\ & -3y & +z & = 2 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

(l) represents and solves linear and nonlinear equations and inequalities graphically

Problem 2.11. • *Graph the solution to the following simultaneous inequalities.*

$$\begin{aligned} 2x + y &> 2 \\ 3x + 5y &\leq 3 \end{aligned}$$

• *Find the maximum of* $f(x) = 2x + 4y$ *subject to* $x > 0$, $y > 0$, $2x + y \leq 4$ *and* $-3x + y \leq 5$.

3 Competency 0003: Functions

4 Competency 0004: Calculus

5 Competency 0005: Geometry and Measurement

6 Competency 0006: Statistics and Probability

7 Competency 0006: Pedagogical Content Knowledge