Math 3330 - Test 2 Review

1 Paths and Vector Functions

1. Let $\mathbf{r}(t) = \langle t, t^2 \rangle$. Graph this function. Find the tangent line to $\mathbf{r}(t)$ at the point $t = 1$ and graph the tangent line as well.

2. Let $\mathbf{r}(t) = \langle \cos(3t), \sqrt{2}\cos(3t), -\cos(3t), 2\sin(3t) \rangle$. Find the tangent line to $\mathbf{r}(t)$ at the point $t = \pi$.

3. Let $\mathbf{r}(t) = \langle \cos(3t), \sqrt{2}\cos(3t), -\cos(3t), 2\sin(3t) \rangle$. Compute the arclength from $t = 0$ to $t = \pi/2$.

4. Let $\mathbf{r}(t) = \langle \cos(3t), \sqrt{2}\sin(3t) \rangle$. Compute the velocity and acceleration at the points $t = 0$ and $t = \pi/2$. Graph the acceleration and velocity coming from the point.

2 Partial Derivatives

5. Sketch the contour plot for $f(x, y) = x^2 - 2y^2$.

6. At each of the points draw the cooresponding gradient vector.

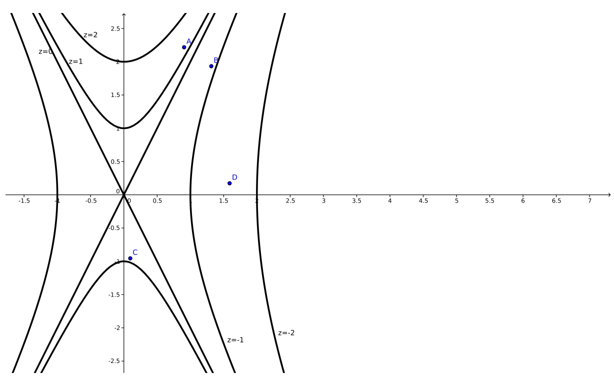

Figure 1:

7. Compute the following limits if they exist. If not show why.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 + y^3}{x^2 + y^2 + 1}$

(b) $\lim_{(x,y)\to(0,0)} \dfrac{1-\cos(x^2+y^2)}{x^2+y^2}$

(c) $\lim_{(x,y)\to(0,0)} \dfrac{x^2+xy+y^2}{x^2+y^2}$

8. Let $f(x, y) = x^2 - 2y^2$. Find the tangent planes at $P(1, 2)$. Use the tangent plane to approximate $f(.9, 1.9)$.

9. Let $f(x, y) = e^{x^2+y^3}$. Find the tangent planes at $P(1, -1)$. Use the tangent plane to approximate $f(0.9, -0.9)$.

10. Let $f(x, y, z) = \sin(x^3 - 2y^2 z) + x^2 y$. Find the tangent planes at $P(2, -2, 1)$. Use the tangent plane to approximate $f(2, -1.9, 1.1)$.

11. Compute the gradient of $f(x, y) = e^{x^2+y^3}$. And compute the directional derivative of $f(x, y)$ at the point $P(1, 1)$ in the direction of $\langle 1, -4\rangle$.

12. Compute the gradient of $f(x, y) = e^{x^2+y^3}$. And compute the directional derivative of $f(x, y)$ at the point $P(1, 1)$ in the direction of maximum increase.

13. Use the second derivative test to find and classify the extremma for:

    (a) $f(x, y) = x^3 - 3x^2 + y^3 - y + 3$

    (b) $f(x, y) = x^3 + x^2 y + y^3 - 9y - 3$

14. Use the LaGrange Multipliers to find max/min for:

    (a) $f(x, y, z) = x^2 + y^2 + z^2$ subject to $x + y - 2z = 1$

    (b) $f(x, y, z) = x + y - 2z$ subject to $x^2 + y^2 + z^2 = 1$

    (c) $f(x, y, z) = x\ln(x) + y\ln(y) + z\ln(z)$ subject to $x + y + z = 1$

3 Double Integrals

15. $\iint_R x + y\, dA$ over the region defined by $x + y = 2$ and the coordinate axes.

16. $\iint_R xy\, dA$ over the region defined by $y = x^2$ and the line $y = x + 1$.

17. $\iint_R e^{x^2}\,dA$ over the region defined by $y = -x$, $y = 2x$ and the vertical line $x = 4$.

18. $\iint_R e^{x^2+y^2}\,dA$ over the region defined by the portion of the circle $x^2 + y^2 = 4$ in the third quadrant.

19. $\iint_R \sqrt{\dfrac{\tan^{-1}(y/x)}{x^2+y^2}}\,dA$ over the region defined by the portion of the circle $x^2 + y^2 = 4$ above the lines $y = -x$ and $y = x$.

20. Find the volume below the paraboloid $z = 12 - x^2 - y^2$ and above the xy-plane.

21. $\iint_R \sin(x-y)\cos(x+y)\,dA$ over the region defined the lines $y = x+2$, $y = x + 4$, $y = -x$ and $y = -x + 3$. Hint the change of variables is $u = x - y$ and $v = x + y$.

22. $\iint_R \dfrac{x-y}{2x+y}\,dA$ over the region defined the lines $y = x + 2$, $y = x$, $y = -2x + 2$ and $y = -2x + 3$.

23. $\iint_R xy\,dA$ over the region defined the graphs of $xy = 1$, $xy = 3$ and the lines $y = x$ and $y = 3x$. Hint $x = u/v$ and $y = v$.

24. $\iint_R (x-y)e^{x^2-y^2}\,dA$ over the region defined the lines $y = x+2$, $y = x$, $y = -x$ and $y = -x + 3$.

25. $\iint_R e^{x^2+4y^2}\,dA$ over the region defined by the portion of the ellipse $\frac{x^2}{2} + y^2 = 1$ in the third quadrant. Hint use the change of variables $x = 4v\cos(u)$ and $x = v\sin(u)$. And note I had $0 \le u \le 2\pi \ldots$