Math 3330 - Some More Review

1. Find the vector $\mathbf{v} \in \mathbb{R}^2$ so that $\mathbf{v}$ is 3-units long and if its initial point is the origin, $\mathbf{v}$ makes a 135% angle with the x-axis.

2. Let $\mathbf{v_1}, \mathbf{v_2} \in \mathbb{R}^3$ so $\mathbf{v_1} = \langle 1, 2, 3\rangle$ and $\mathbf{v_2} = \langle 2, -1, 1\rangle$. Find c_1 and c_2 so that
$$c_1\mathbf{v_1} + c_2\mathbf{v_2} = \langle 4, -7, -3\rangle.$$

3. Let $\mathbf{v_1}$, $\mathbf{v_2}$ and $\mathbf{v_3} \in \mathbb{R}^3$ so $\mathbf{v_1} = \langle 1, 2, 3\rangle$ $\mathbf{v_2} = \langle 2, -1, 1\rangle$ and $\mathbf{v_3} = \langle 4, -3, 7\rangle$.
   (a) Find angle between $\mathbf{v_1}$ and $\mathbf{v_2}$.
   (b) Find three different unit vectors perpendicular to $\mathbf{v_1}$.
   (c) Find $\mathbf{u}$ a unit vector parallel to $\mathbf{v_3}$.
   (d) Compute $\mathbf{u} \cdot \mathbf{v_3}$ and compute $\|\mathbf{v_3}\|$.

4. Let $\mathbf{v_1}$, $\mathbf{v_2}$ and $\mathbf{v_3} \in \mathbb{R}^3$ so $\mathbf{v_1} = \langle 1, 2, 3\rangle$ $\mathbf{v_2} = \langle 2, -1, 1\rangle$ and $\mathbf{v_3} = \langle 4, -3, 7\rangle$.
   (a) Compute $\mathbf{v_1} \times \mathbf{v_2}$.
   (b) Compute the area of the parallelogram formed by the two vectors $\langle 2, -1\rangle$ and $\langle 4, -3\rangle$. Also draw the vectors and the parallelogram.
   (c) Compute the volume of a parallelepiped formed by the three vectors $\mathbf{v_1}$, $\mathbf{v_2}$ and $\mathbf{v_3}$.
   (d) Find a single unit vector $\mathbf{w}$ that is simultaneously perpendicular to $\mathbf{v_1}$ and $\mathbf{v_2}$.
   (e) Show that any vector of the form $\langle a - 3b, a, b - a\rangle$ where $a, b \in \mathbb{R}$ is perpendicular to $\mathbf{v_1}$.

5. Find the equation of the plane containing the points $P(1, 2, 3)$, $Q(1, 0, 1)$ and $R(9, 0, 1)$.

6. Find the equation of a line (any line) contained within the plane $x - y + z = 2$.

7. Find the equation of a line containing the point $(1, 2, 3)$ perpendicular to the plane $x - y + z = 2$.

8. Find the equation of the plane containing the lines

$$L_1 : \begin{cases} x = & 2 - 2t \\ y = & 3 + t \\ z = & 3 + 4t \end{cases} \quad L_2 : \begin{cases} x = & 2 - 2t \\ y = & 3 + 2t \\ z = & 3 \end{cases}$$

9. For the two planes below show they do not intersect.

$$P_1 : x + y - z = 4 \ P_2 : x + y - z = 5$$

10. For the two planes below find the equation of the line intersecting

$$P_1 : x + y - z = 4 \ P_2 : 2x + -3y + z = 4$$

11. For the two planes below find the angle of intersection.

$$P_1 : x + y - z = 4 \ P_2 : 2x + -3y + z = 4$$