Math 2320 - Test 2 Review

0 A list of the series tests

- Telescoping Series
- Geometric Series
- Divergence Test
- Integral Test
- P-series
- Comparison Test
- Limit Comparison Test
- Ratio Test
- Root Test
- Alternating Series Test

1 Integrals

1. $\int xe^{x^2+7}\,dx$
2. $\int xe^{3x}\,dx$
3. $\int \frac{1}{\sqrt{4+x^2}}\,dx$
4. $\int \frac{1}{(9-x^2)^{3/2}}\,dx$
5. $\int \frac{1}{(x^2-1)^{1/2}}\,dx$
6. $\int \frac{1}{(x^3-x)}\,dx$ use partial fractions

7. $\int \frac{2x^2 - x - 1}{(x^3 + x)}\, dx$

8. $\int \frac{x^3}{x^2 - 1}\, dx$ note the polynomial fraction is improper

9. $\int_0^\infty xe^{-x^2}\, dx$

10. $\int_1^\infty \frac{1}{x^3}\, dx$

11. $\int_1^\infty \frac{1}{\sqrt{x}}\, dx$

2 Series

12. $\sum_{n=1}^{\infty} \frac{1}{2n} - \frac{1}{2n+2}$

13. $\sum_{n=2}^{\infty} 2 \cdot 3^n$

14. $\sum_{n=-1}^{\infty} \frac{2}{3^n}$

15. $\sum_{n=1}^{\infty} \frac{2n^2 + 3n - 1}{5n^3 + 1}$

16. $\sum_{n=1}^{\infty} ne^{-n^2}$ Use integral test

17. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

18. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

19. $\sum_{n=1}^{\infty} \frac{n^2 + 1}{\sqrt{n^3 + 1}}$

20. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{\sqrt{n^5+1}}$

21. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

22. $\displaystyle\sum_{n=11}^{\infty} \frac{1}{n^2-1}$

23. $\displaystyle\sum_{n=3}^{\infty} \frac{n}{n^2-1}$

24. $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$ Use ratio test

25. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!}$

26. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3^n}$

27. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

28. $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

29. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

30. $\displaystyle\sum_{n=1}^{\infty} \left[\frac{2n^2+3n-1}{5n^3+1}\right]^n$

31. $\displaystyle\sum_{n=1}^{\infty} \left[1-\frac{1}{n}\right]^{n^2}$

32. $\displaystyle\sum_{n=1}^{\infty} \left[\frac{\sin(n)}{2n}\right]^n$

33. $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2+3n-1}{5n^3+1}(-1)^n$

34. $\displaystyle\sum_{n=1}^{\infty} \left(1-\frac{2}{n}\right)^n (-1)^n$

35. $\displaystyle\sum_{n=1}^{\infty} \left(1-\frac{2}{n}\right)^{n^2} (-1)^n$

3 Power Series and Taylor Polynomials

36. Compute the interval of convergence and radius of convergence for the following series

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{3^n}$

    (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+3)^n}{3^n}$

    (c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$

    (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-4)^n}{n^2+1}$

37. Compute the Taylor Series for the following functions.

    (a) $f(x) = \sin(3x)$ at $a = 0$

    (b) $f(x) = e^{5x}$ at $a = 0$

    (c) $f(x) = e^{3x}$ at $a = 1$

    (d) $f(x) = \sin(3x)$ at $a = \pi$

    (e) $f(x) = \cos(x)$ at $a = 0$

    (f) $f(x) = \sin(x)$ at $a = 0$

    (g) $f(x) = e^x$ at $a = 0$

    (h) $f(x) = \frac{1}{1-x}$ at $a = 0$

    (i) $f(x) = x^3 + 2x - 1$ at $a = 1$

38. Use calculus-type and algebraic manipulations to the given Taylor series to compute the unknown Taylor series. Recall (and memorize) the Taylor series (with $a = 0$) following:

- $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^5 + x^n \cdots$
- $e^x = 1 + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 - \cdots + \frac{1}{n!}x^n \cdots$
- $\sin(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \cdots + \frac{(-1)^n}{(2n+1)!}x^{2n+1} \cdots$
- $\cos(x) = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 - \cdots + \frac{(-1)^n}{(2n)!}x^{2n} \cdots$

(a) $f(x) = \sin(5x)$ with $a = 0$

(b) $f(x) = x^2 \sin(x^2)$ with $a = 0$

(c) $f(x) = \frac{1}{1+x^2}$ with $a = 0$

(d) $f(x) = \tan^{-1}(x)$ with $a = 0$