5.1: ※17, ※35

(17.) $f(x) = x^2 - 1$ ON $[2,4]$; $n=4$

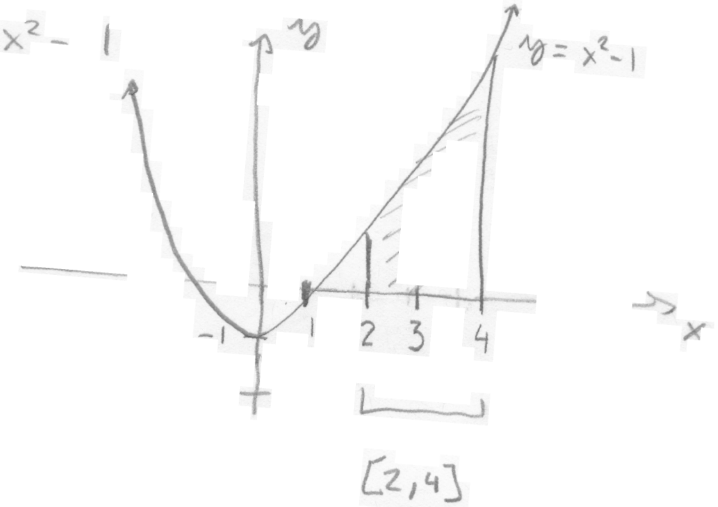
(a) Sketch

(b) calculate Δx AND $x_0, x_1, \ldots x_n$

(c) ILLUSTRATE RIGHT RIEMANN SUMS

(d) CALCULATE RIGHT RIEMANN SUMS

(a) $y = x^2 - 1$



(b) CALCULATE $\Delta x = \frac{b-a}{n} = \frac{4-2}{4} = \frac{1}{2}$ $\quad x_k = a + k\Delta x = 2 + k \cdot \frac{1}{2}$

$x_0 = 2 + 0 \cdot \frac{1}{2} = 2$

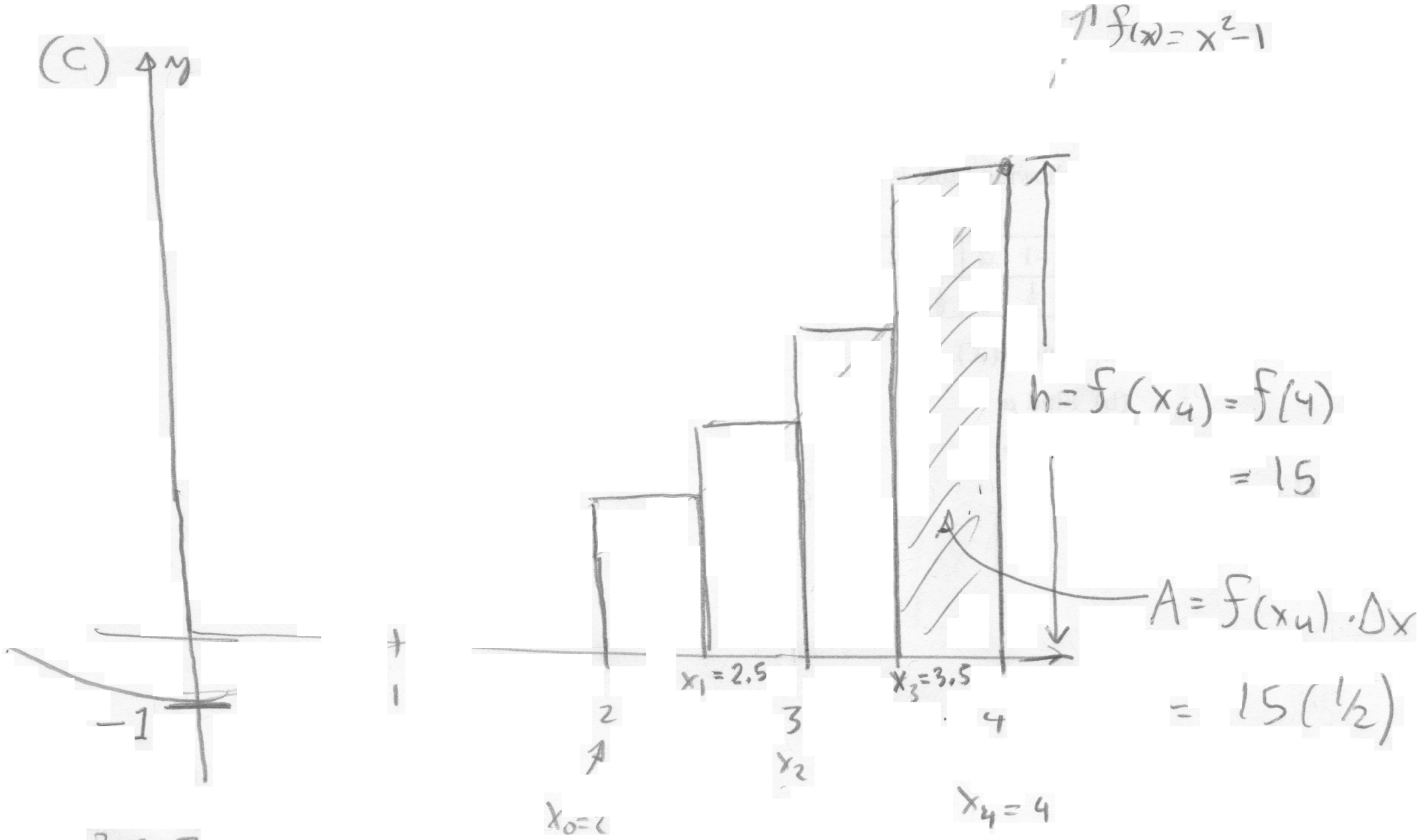
$x_1 = 2 + 1 \cdot \frac{1}{2} = 5/2$

$x_2 = 2 + 2(1/2) - 3$

$x_3 = 2 + 3(1/2)\ 7/2$

$x_4 = 2 + 4(1/2) - 4$

(c)



(d) RIGHT RIEMANN SUM

$$= f(x_1)\cdot\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\cdot\Delta x$$

$$= f\left(\tfrac{5}{2}\right)\cdot\tfrac{1}{2} + f(3)\tfrac{1}{2} + f(7/2)\cdot\tfrac{1}{2} + f(4)\cdot\tfrac{1}{2}$$

$$= \left(\left(\tfrac{5}{2}\right)^2-1\right)\cdot\tfrac{1}{2} + (3^2-1)\cdot\tfrac{1}{2} + \left(\left(\tfrac{7}{2}\right)^2-1\right)\cdot\tfrac{1}{2} + (4^2-1)\cdot 1/2$$

$$= \frac{79}{4} = 19.75$$

35. $f(x) = \sqrt{x}$ $\quad [0,4] \quad \underbrace{n = 40}$

For $n=40$ we may not want to add up 40 terms in the sum. So we can use technology to help.

$f(x) = \sqrt{x}$ $\quad a=0, \quad b=4, \quad n=40$

$$\Delta x = \frac{b-a}{n} = \frac{4-0}{40} = \frac{1}{10}$$

$$x_k = a + k\cdot\Delta x = 0 + k\cdot\frac{1}{10} = \frac{k}{10}$$

RIGHT RIEMAN SUM $= \displaystyle\sum_{k=1}^{40} f(x_k)\,\Delta x = \sum_{k=1}^{40} \sqrt{\frac{k}{10}}\cdot\frac{1}{10}$

$$= \frac{1}{10\sqrt{10}} \sum_{k=1}^{40}\sqrt{k} = \frac{1}{10\sqrt{10}}\left[\sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} + \cdots + \sqrt{40}\right]$$

Here use a calculator, computer or a formula

$$= \boxed{5.427}$$

you now know FTC II. Compute the true value.

$$\int_0^4 \sqrt{x}\,dx = \frac{2}{3}x^{3/2} = \boxed{5\frac{1}{3}}$$