

Math 4100 Quiz 2

Name: _____

1. There were 63 equal piles of plantain plus 7 individual fruits. The fruit was divided evenly among 23 people. How many pieces of fruit were there?
2. A farmer purchased 100 head of livestock for 4000.00. The prices were as follows \$120 for a cow \$50 for a lamb and \$25 for a piglet. Find out how many of each he bought.
3. Prove any integer of the form $3n + 2$ has a prime factor of the form $3m + 2$. (3.1.3b)
4. Prove the only prime, p , for which $3p + 1$ is a perfect square is $p = 5$. (3.1.3d)
5. Prove there are infinitely many primes.
6. Prove $\sqrt{5}$ is irrational.
7. Let p be a prime and assume $p \nmid b$. Show that in the following progression that every p th term is divisible by p :

$$a, a + b, a + 2b, a + 3b, a + 4b, a + 5b, \dots$$

Hint: Use $\gcd(p, b) = 1$ to get $ax + by = 1$ show p divides $a + (kp - ay)b$ where k is any integer. (3.2.18)

8. If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then $a + c \equiv b + d \pmod{n}$.
9. If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then $ac \equiv bd \pmod{n}$.
10. Assume $a^2 \equiv b^2 \pmod{n}$. Show by a counterexample that $a \equiv b \pmod{n}$ need not be true.
11. Show $39 \mid 53^{103} + 103^{53}$.
12. (4.2.12)
13. (4.3.9)
14. (4.3.10)