Math 4100 Quiz 1

Name:____

1. Prove with induction that for all $n \in \mathbb{N}$ we have
$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

2. Prove with division algorithm that for all $n \in \mathbb{Z}$ we have $\frac{n(n+1)(2n+1)}{6}$ is always an integer.

3. Prove if $a|b$ and $a|c$ then $a^2|bc$.

4. Prove with induction that for all $n \in \mathbb{N}$ we have $15|2^{4n} - 1$.

5. The product of four consecutive integers is one less than a perfect square. (2.3.8b).

6. Confirm the following properties of the GCD:

   (a) $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$ implies $\gcd(a, bc) = 1$.

   (b) $\gcd(a, b) = 1$ implies $\gcd(ac, b) = \gcd(c, b)$.

7. Use the Euclidean Algorithm to obtain x and y so that

   (a) $\gcd(56, 72) = 56x + 72y$

   (b) $\gcd(119, 272) = 119x + 272y$

8. Assume that $\gcd(a, b) = 1$. Show $\gcd(a + b, a - b) = 1$ or 2.
   Recall: If $\gcd(A, B) = d$ then $d|Ax + By$ for any $x, y \in \mathbb{Z}$. So in particular $d|A + B$ and $d|A - B$.

9. Prove $\gcd(a, b)$ divides $\text{lcm}(a, b)$.