Math 3160 - Practice Test 2

Name:_____

1. Consider the system of linear equations.

$$\begin{aligned} x_1 + x_2 + x_3 &= 3 \\ x_2 + 5x_3 &= 1 \\ x_1 + x_3 &= 7 \end{aligned}$$

   (a) Write the system as a matrix multipication $A\mathbf{x} = \mathbf{b}$

   (b) Solve by finding A^{-1}.

2. Use Cramer's Rule to solve.

$$\begin{array}{rrrl} 2x_1 & +4x_2 & +x_3 & = 3 \\ & 5x_2 & +2x_3 & = 0 \\ x_1 & + & +x_3 & = 8 \end{array}$$

3. Define the following vectors

$$\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 3 \\ 6 \\ 6 \\ 6 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 3 \end{bmatrix}$$

(a) Is the list of vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ linearly independent?

(b) Is the vector $\mathbf{b}$ in the span of $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$?

4. Define the following $A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ -1 & 2 & 1 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{bmatrix}$ $\mathbf{c} = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

   (a) A is a transformation from $\mathbb{R}^n \to \mathbb{R}^m$. Find n, m. What is the dimension of the domain?

   (b) Is $\mathbf{c} \in$ COL(A)?

   (c) Compute the column space for A, Write as the span of a basis.

(d) Compute the null space for A, Write as the span of a basis.

(e) Compute Rank and Nullity of A. Compare to your answer for 4a.

5. Solve for any $\mathbf{x}$ so that $A\mathbf{x} = \mathbf{x}$ where $A = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$ $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.