Math 3160 - Practice Test 2

1. Solve the system of linear equations. $\begin{array}{rrrl} x_1 & +x_2 & +x_3 & = 3 \\ & x_2 & +5x_3 & = -13 \\ -2x_1 & +3x_2 & +x_3 & = -5 \end{array}$

2. Compute the following matrix computations.

   $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 1 \\ 0 & 2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} a & b \\ c & d \\ d & e \end{bmatrix}$

   (a) AB.

   (b) BA.

   (c) $\det(A)$.

   (d) $\det(B)$.

3. Solve the following for x and y.

   $\begin{array}{lll} 3x^3 & +y^2 & = 1 \\ x^3 & +2y^2 & = 7 \end{array}$

4. Assume A is an $n \times n$ matrix that satisfies

   $$A^3 - A^2 + A + I = 0$$

   Show $A^{-1} = -I + A - A^2$.

5. Solve for any $\mathbf{x}$ so that $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ $\mathbf{b} = \begin{bmatrix} 3 \\ 11 \end{bmatrix}$

   (a) Solve with row reduction.

   (b) Solve by finding A^{-1}.

6. Solve for any $\mathbf{x}$ so that $A\mathbf{x} = \mathbf{x}$ where $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

7. Use Cramer's Rule to solve. $\begin{array}{rrrl} x_1 & +x_2 & +x_3 & = 3 \\ & x_2 & +5x_3 & = -13 \\ -2x_1 & +3x_2 & +x_3 & = -5 \end{array}$

8. Define the following vectors

   $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 3 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} -1 \\ 2 \\ 2 \\ 2 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 5 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 3 \end{bmatrix}$

   (a) Is the list of vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ linearly independent?

   (b) Is the vector $\mathbf{b}$ in the span of $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$?

9. Define the following $A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 2 & 1 & 1 \\ 0 & 2 & 0 & 1 \end{bmatrix}$ $\mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 3 \end{bmatrix}$ $\mathbf{c} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$

   (a) A is a transformation from $\mathbb{R}^n \to \mathbb{R}^m$. Find n, m. What is the dimension of the domain?

   (b) Find the image of $\mathbf{b}$ after applying the transformation A

   (c) Is $\mathbf{c} \in$ COL(A)?

   (d) Compute the column space for A, Write as the span of a basis.

   (e) Compute the null space for A, Write as the span of a basis.

   (f) Compute Rank and Nullity of A. Compare to your answer for 9a.

10. Compute the determinate of the matrix below given that $det(A) = 10$ where

$$A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$$

   (a) $B = \begin{bmatrix} a & b & c & d \\ a & b & c & d \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$

   (b) $C = \begin{bmatrix} a & b & c & d \\ a+e & b+f & c+g & d+h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$

   (c) $C = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ 3i & 3j & 3k & 3l \\ m & n & o & p \end{bmatrix}$

   (d) $D = 3A$