Name:________________________________

MA 5230 Test 2

1. Use the following data for this exercise:

$$2, 2, 3, 3, 5$$

   (a) Find $\bar{x}$ and s.
   (b) Compute the 95% confidence interval.
   (c) Use $H_0 : \mu = 4.0$ as your null hypothesis and run the hypothesis test.
   (d) Compute the P-Value for $\mu = 4.0$.

(a) $\bar{X} = \frac{2+2+3+3+5}{5} = \frac{15}{5} = \boxed{3}$

$S^2 = \frac{1}{n-1} \sum_i (x_i - \bar{x})^2 = \frac{1}{5-1}\left[(2-3)^2 + (2-3)^2 + \cdots + (5-3)^2\right]$

$= \frac{1}{4}\left[1^2 + 1^2 + 0^2 + 0^2 + 4\right] = \frac{6}{4} = \frac{3}{2} = 1.5$

$\boxed{s = \sqrt{1.5}}$

(b) $SE = \frac{s}{\sqrt{n}} = \frac{\sqrt{1.5}}{\sqrt{5}} = \sqrt{0.3} = .5477$

$m = t^* SE = (2.776)(.5477) = 1.52$

t-dist dF = 4
P = 0.025

$\bar{x} \pm m$

$3 \pm 1.52 = \boxed{[1.48, 4.52]}$

(c) $H_0 : \mu = 4.0$

DO NOT REJECT SINCE 4.0 IS IN 95% CONF. INTERVAL

(d) $t = \frac{4-3}{\sqrt{0.3}} = \frac{1}{\sqrt{.3}} = 1.82$

$\mu = 4$, $\bar{x} = 3$, SE

LIES BETWEEN 10% & 5% ON t dist.
(1.533) (2.132)

So $P = 1 - 2(10\%)$ to $1 - 2(5\%)$

$P =$ 80% to 90%

$P =$ 85.77% ← FROM CALCULATOR

2. Roll a four-sided die 20 times to test the probability that 1 comes up with $\frac{1}{4}$ probability. So our null hypothesis is

$$H_0 : p = \frac{1}{4}.$$

We will reject the null hypothesis if we get $N < 3$ or $N > 7$ where N is the number of 1's. H_0: $p = \frac{1}{4}$. H_a: $p = \frac{1}{2}$.

(a) Compute the percentage confidence of our interval.

(b) Compute the Type I error.

(c) Compute the Type II error.

(a) $P(p=\frac{1}{4}$ AND $3 \leq N \leq 7)$

FROM BINOMIAL TABLE $n=20$ $p=0.25$

$= P(N=3) + P(N=4) + \cdots + P(N=7)$

$= .1339 + .1897 + .2023 + .1686 + .1124$

$= .8069$ $\boxed{80.69\% \text{ CI}}$

(b) $\alpha = 1 - P = 1 - 80.69\% = \boxed{19.31\%}$

(c) $\beta = P(p = \frac{1}{2}$ AND $3 \leq N \leq 7)$

H_a

FROM BINOMIAL TABLE $n=20$ $p=.50$

$= P(N=3) + \cdots + P(N=7)$

$= .0011 + .0046 + .0148 + .0370 + .0739$

$= .1314$ $\boxed{\beta = 13.14\%}$

3. We will rerun the experiment from Problem 2. Assume we roll the die 20 times and we get the following result

Face showing	1	2	3	4
Number of rolls	6	8	4	2

Run the Chi squared test to test the hypothesis that the die is fair.

OBSERVED

$\frac{20}{4} = 5$

EXPECTED

1	2	3	4
5	5	5	5

$$\chi^2 = \sum \frac{(OBS - EXP)^2}{EXP} = \frac{(6-5)^2}{5} + \frac{(8-5)^2}{5} + \frac{(4-5)^2}{5} + \frac{(2-5)^2}{5}$$

$$= 4$$

$\underline{\underline{H_0}}$: DIE IS FAIR

$dF = n-1 = 4-1 = 3$

95% CI 0 to 7.81

$4 \in [0, 7.81]$ so we do not reject.

4. Consider the following data.

	number	mean	standard deviation
1	$n_1 = 15$	$\bar{x}_1 = 7$	$\sigma_1 = 11$
2	$n_2 = 13$	$\bar{x}_2 = 3$	$\sigma_2 = 9$

Test the hypothesis that $\mu_1 = \mu_2$.

$H_0: \mu_D = 0$ where $D = X_1 - X_2$

$\bar{X}_D = \bar{X}_1 - \bar{X}_2 = 7 - 3 = 4$

$SE_D = \sqrt{\frac{11^2}{15} + \frac{9^2}{13}} = 3.78$

95% NORMAL

(why NORMAL?, WHY NOT t?)

$m = z^* SE = 1.96(3.78) = 7.41$

CI $\bar{X} \pm m = 4 \pm 7.41$

-3.41 to 11.41

$\mu_D = 0$ is NULL HYPOTHESIS (DO NOT REJECT).

5. Assume we flip a coin 40 times and observe 25 heads. We will examine the question of the fairness of the coin?

(a) Compute a 95 % confidence interval.

(b) Do you accept or reject the hypothesis (what is your null hypothesis)?

(c) Compute the p-Value.

(a) $\hat{p} = \frac{25}{40} = 62.5\%$

$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{.625\,(1-.625)}{40}} = 7.65\%$

$m = z^{*} \cdot SE = 1.96(7.65\%) = 15.0\%$

$62.5\% \pm 15.0\%$

$\boxed{47.5\% \text{ to } 77.5\%}$

(b) $H_0 : P = \frac{1}{2}$

DO NOT REJECT SINCE .50 IS IN CI.

(c) $P = \frac{1}{2}$ $\qquad \frac{\frac{1}{2} - .625}{SE} = \frac{12.5\%}{7.65\%} = 1.63$

.9484 ← FROM STD NORMAL TABLE

So $P = 1 - 2 \cdot (5.16\%)$

$\boxed{P = 89.68\%}$

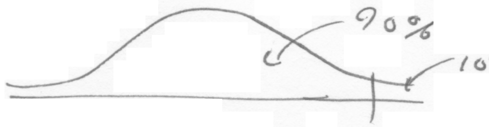
$1 - .9484 = 5.16$

6. This question refers to the Problem 5

(a) For the same coin flipping experiment (40 coin flips with Null Hypothesis $p = 1/2$) compute a 90 % one-sided confidence interval.

(b) Redesign the original coin flipping experiment where we want the margin for the 95 % Confidence Interval to be $m = 0.010$. Find the number of coin flips needed.

(a)



$Z = 1.29$ ← FROM TABLE

$m = z^* SE$

$= 1.29\,(7.65\%)$

$= 9.87\%$

0 to $\bar{X} + 9.87\%$

0 to $62.5\% + 9.87\%$

$\boxed{0 \text{ to } 72.4\%}$

(b)

$$n = \frac{1}{4}\left(\frac{z^*}{m}\right)^2 = \frac{1}{4}\left(\frac{1.96}{0.010}\right)^2 = \boxed{9604}$$